Physics 786, Spring 2023
Problem Set 2, due Friday, February 17.

1. *Gravitational Twin Paradox*

Suppose you and a friend synchronize your perfect watches, and then you move to the 102nd floor of the Empire State Building while your friend stays the ground. How long would you have to wait in order for the time measured on your watches to differ by one second? Whose watch is ahead of the other?

2. *Gravitational Redshift*

Suppose a radio signal from a GPS satellite is sent to the ground. If the satellite is 20,200 km above Earth's surface, by what fraction is the frequency of the radio wave increased *due to gravity* compared to the emitted frequency when observed on the ground?

3. *Geodesics in Scalar Gravity*

Suppose the metric took the form $g_{\mu\nu} = \eta_{\mu\nu}\left(1 + 2\phi(\mathbf{x}, t)\right)$, where ϕ is the gravitational potential.

a) If $\phi = gz$, where g is the acceleration of gravity near the earth and z is the vertical displacement from the ground, what are the nonvanishing components of the Christoffel symbols $\Gamma^{\mu}_{\nu\lambda}$?

b) In the Newtonian approximation, use the geodesic equation to calculate the acceleration of a freely falling massive particle $\frac{d^2\mathbf{x}}{dt^2}$.