Physics 786, Spring 2017
Problem Set 8, Due Thursday, April 20, 2017.

1. *Binary Black Hole Systems*

Assume the two black holes in a binary black hole system each have mass M=30 solar masses, with orbital period beginning at T=0.01 s, when the orbital radius about the center of mass is R=140 km.

a) Assuming circular orbits, treating the system in a Newtonian approximation find the power radiated in gravitational radiation in Watts.

b) In the same system, find the change in the orbital period after each complete orbit.

2. *The Kerr spacetime*

In this problem you will consider the rotating black hole metric, discovered by Roy Kerr in 1963. The metric can be written in Boyer-Lindquist form as

$$\begin{aligned} ds^2 &= -\left[1 - \frac{2GMr}{r^2 + a^2\cos^2\theta}\right] dt^2 - \frac{4GMr\, a \sin^2\theta}{r^2 + a^2\cos^2\theta} dt\, d\phi \\ &+ \left[\frac{r^2 + a^2\cos^2\theta}{r^2 - 2GMr + a^2}\right] dr^2 + (r^2 + a^2\cos^2\theta) d\theta^2 \\ &+ \left[r^2 + a^2 + \frac{2GMr\, a^2 \sin^2\theta}{r^2 + a^2\cos^2\theta}\right] \sin^2\theta\, d\phi^2, \end{aligned}$$

where a is a constant related to the angular velocity of the black hole if $M \neq 0$.

If $M = 0$ then the metric reduces to the Minkowski metric in oblate spheroid coordinates, related to usual Cartesian coordinates by

$$\begin{aligned} x &= \sqrt{r^2 + a^2} \sin\theta \, \cos\phi \\ y &= \sqrt{r^2 + a^2} \sin\theta \, \sin\phi \\ z &= r\cos\theta. \end{aligned}$$

a) Show that if a=0 then the metric reduces to the Schwarzschild metric.

b) For what relations between r and θ are components of the metric divergent? There should be several such relations.

c) The singularity at $r = 0$ with $\theta = \pi/2$ is a curvature singularity. The other metric singularities correspond to the horizons. What is the shape of the curvature singularity in terms of the coordinates x, y and z as defined above?

d) Suppose an observer uses a jet pack to stand still, with constant r, θ and ϕ in the Boyer-Lindquist coordinates. The tangent vector of the observer is $v^\mu \equiv dX^\mu/dt = (1, 0, 0, 0)$. This describes a physical tangent vector only if v^μ is timelike, *i.e.* $v^\mu v^\nu g_{\mu\nu} < 0$. Find the relation(s) between r and θ such that $v^\mu v^\nu g_{\mu\nu} = 0$.

These regions are called ergosurfaces. In the region (called the ergosphere) outside both horizons and inside the outer ergosurface, observers cannot stand still, but are dragged around the black hole. Roger Penrose suggested that energy could be taken from a rotating black hole by sending objects through this region, with part of the objects falling into the black hole and the other parts propelled away from the black hole.

You should compare your results with Fig. 1 in Matt Visser's review at https://arxiv.org/pdf/0706.0622.pdf.