Physics 786, Spring 2017
Problem Set 6, Due Thursday, March 23, 2017.

1. *Harmonic Coordinates*

Show that the harmonic coordinate conditions $g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu} = 0$ are equivalent to the conditions

$$\frac{\partial}{\partial x^{\mu}}\left(\sqrt{g}\, g^{\mu\lambda}\right) = 0.$$

2. *2D Anti-de Sitter Spacetime*

Consider the 2D Anti-de Sitter spacetime with metric

$$ds^2 = a^2\left(-\cosh^2\rho\, d\tau^2 + d\rho^2\right), \quad a = \text{const.}$$

a) Calculate all the components of the affine connection $\Gamma^{\mu}_{\nu\lambda}$.

b) Calculate all the components of the Ricci tensor $R_{\mu\nu}$ and the curvature scalar R.

c) Show that $R_{\mu\nu\lambda\sigma} = \frac{1}{a^2}\left(g_{\mu\lambda}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\lambda}\right)$.

d) Suppose that the 2D Anti-de Sitter spacetime is the solution to Einstein's equations with some energy-momentum tensor $T_{\mu\nu}$. What is $T_{\mu\nu}$ in terms of a and $g_{\mu\nu}$?

e) Let $r = a\sinh\rho$ and $t = a\tau$. Write the metric in r, t coordinates.

3. *Vacuum Solutions in Three Dimensions*

In this problem we will look for static, isotropic black-holes in three spacetime dimensions.

Assume a metric of the form

$$ds^2 = -e^{2\phi(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2 d\theta^2.$$

a) Calculate the nonvanishing Christoffel symbols.

b) Calculate the components of the Ricci tensor R_{tt}, R_{rr}, and $R_{\theta\theta}$. The other components of $R_{\mu\nu}$ vanish.

c) Solve the vacuum Einstein equations $R_{\mu\nu} = 0$, assuming that the metric approaches the flat metric as $r \to \infty$. Are there any nontrivial black-hole solutions?