Physics 786, Spring 2017
Problem Set 2, Due Thursday, February 9, 2017.

1. *Free fall*

Repeat and complete the derivation from class that along the trajectory of a freely falling particle,

$$\frac{d}{d\tau}\left(g_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\right) = 0.$$

2. *Gravitational Redshift*

Suppose while on the 102nd floor of the Empire State Building you shine a laser towards the ground. By what fraction is the frequency of the laser light increased compared to the emitted frequency when observed on the ground?

3. *Geodesics in Scalar Gravity*

Suppose the metric takes the form $g_{\mu\nu} = \eta_{\mu\nu}(1 + 2\phi(\mathbf{x}, \mathbf{t}))$, where ϕ is the gravitational potential. As discussed in class, this would be a good guess for the form of the metric in scalar gravity.

a) If $\phi = gz$, where g is the acceleration of gravity near the earth and z is the vertical displacement from the ground, what are the nonvanishing components of the Christoffel symbols $\Gamma^\mu_{\nu\lambda}$?

b) In the Newtonian approximation, use the geodesic equation to calculate the acceleration of a freely falling particle $\frac{d^2\mathbf{x}}{dt^2}$.

c) Along a lightlike trajectory $x^\mu(t)$, the geodesics satisfy

$$g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt} = 0.$$

Show that the lightlike trajectories in any potential ϕ are straight lines, as in special relativity. The absence of bending of light in scalar gravity is in contradiction to observation.

4. *Geodesics on the 2-sphere*

In spherical coordinates, the length element on the 2-sphere of radius R takes the form

$$ds^2 = R^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right).$$

a) With $x^1 = \theta$ and $x^2 = \phi$, the metric $g_{ij} = g_{ji}$ is defined such that $ds^2 = g_{ij}dx^i dx^j$, summed over i and j. What are the components of g_{ij}, written as a 2×2 matrix?

b) Find the nonvanishing components of the connection

$$\Gamma^i_{jk} = \frac{1}{2} g^{im} \left(\frac{\partial g_{mj}}{\partial x^k} + \frac{\partial g_{mk}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^m} \right).$$

c) Consider a path parametrized by a parameter t. The paths of shortest distance satisfy the geodesic equation:

$$\frac{d^2 x^i}{dt^2} + \Gamma^i_{jk} \frac{dx^j}{dt} \frac{dx^k}{dt} = 0.$$

Show that arcs along the equator $\theta = \pi/2$ are geodesics on the 2-sphere.