Physics 786, Spring 2014
Problem Set 6, Due Wednesday, April 16, 2014.

1. *Vacuum Solutions in Three Dimensions*

In this problem we will look for static, isotropic black-holes in three space-time dimensions.

Assume a metric of the form

$$ds^2 = -e^{2\phi(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2 d\theta^2.$$

a) Calculate the nonvanishing Christoffel symbols.

b) Calculate the components of the Ricci tensor R_{tt}, R_{rr}, and $R_{\theta\theta}$. The other components of $R_{\mu\nu}$ vanish.

c) Solve the vacuum Einstein equations $R_{\mu\nu} = 0$, assuming that the metric approaches the flat metric as $r \to \infty$. Are there any nontrivial black-hole solutions?

2. *Newtonian Stars*

Consider static, spherically symmetric solutions to Einstein's equations for a fluid with density $\rho(r)$ and pressure $p(r)$, with metric of the form,

$$ds^2 = -e^{2\phi(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right).$$

Assume the fluid is nonrelativistic, and consider the nonrelativistic limit of Einstein's equations for this system. Assume $\phi(0) = \lambda(0) = 0$, and

$$\rho(r) = \rho_0\left(1 - \frac{r}{R}\right)$$

for $r \leq R$, and $\rho(r) = 0$ for $r > R$.

a) Find the spacetime metric for $r < R$ and $r > R$.

b) Find the pressure in the star $p(r)$, such that $p(R) = 0$.

3. *Binary Star Systems*

Assume the two stars in a binary star system each have mass M=1.39 solar masses, with orbital period T=7.75 hrs, and semimajor axis with respect to one of the stars $a = 2R$=1.95$\times 10^6$ km.

a) Assuming circular orbits find the power radiated in gravitational radiation in Watts.

b) In the same system, find the change in the orbital period after each complete orbit.