Physics 786, Spring 2014
Problem Set 6, Due Wednesday, April 2, 2014.

Final Paper Assignment - Due last day of class

For your final paper, you should identify and describe either a theoretical aspect of or extension of general relativity, or an experiment or application which either makes use of general relativity or provides a test of general relativity. You should explain in detail those aspects of general relativity which are relevant. Example topics include gravitation in extra dimensions, the Cosmic Microwave Background, inflation, Gravity Probe B, LIGO, LISA, WMAP, Planck, BICEP, and GPS devices. As a guideline, aim for 5 double-spaced pages.

Due Wednesday, April 2:

1. *Killing Vectors*

a) Consider 2D Euclidean space in Cartesian coordinates x, y. Find the Killing vectors related to translations and rotation about $x = y = 0$ in these coordinates.

b) What are the corresponding "constants of the motion" and their physical interpretation?

2. *Schwarzschild Spacetime*

a) A massive test particle is released from $r = R > 2GM$ in the Schwarzschild geometry (in standard coordinates), and falls radially inward. Show that the following correctly parametrizes the trajectory:

$$r = \frac{R}{2}(1 + \cos\eta)$$
$$\tau = \frac{R}{2}\left(\frac{R}{2GM}\right)^{1/2}(\eta + \sin\eta).$$

b) Show that the proper time elapsed when the particle reaches $r = 2GM$ is finite.

3. *More Schwarzschild*

A massive test particle is at $r = r_0 < 2GM$ at $t = 0$.

a) Show that

$$\left|\frac{dr}{d\tau}\right| \geq \sqrt{\frac{2GM}{r} - 1}.$$

b) Show that the test particle necessarily reaches $r = 0$ in finite proper time.

4. *The Photon Sphere*

In class we studied the unbound trajectories of photons in the Schwarzschild spacetime. However, it is also possible for light to be bound in closed orbits.

a) Find the radius of circular orbits (*i.e.* the value of r in standard coordinates) in terms of the black-hole mass. The collection of circular orbits is called the photon sphere.

b) In standard coordinates, what is $d\phi/dt$ in the circular orbit with $\theta = \pi/2$?

5. *The Planck Mass*

For a particle of mass M, quantum mechanics is important at distance scales of order the Compton wavelength of the particle. What is the condition on M so that the reduced Compton wavelength of the particle, $\lambda_C = \hbar/(Mc)$, is smaller than the Schwarzschild radius of the particle? Compare with the Planck mass, $M_{\rm Pl} = \sqrt{\hbar c/G}$.