Physics 786, Spring 2014
Problem Set 5, Due Wednesday, March 19, 2014.

1. *Curvature of the Two-Sphere*

Consider the two-sphere with metric

$$ds^2 = a^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right).$$

a) Calculate all the components of the affine connection $\Gamma^\mu_{\nu\lambda}$.

b) Calculate all the components of the Ricci tensor $R_{\mu\nu}$ and the Gaussian curvature, $K = -R/2$, of the two-sphere.

Hint: In 2D, $R_{\lambda\mu\nu\rho} = \frac{1}{2}R(g_{\lambda\nu}g_{\mu\rho} - g_{\lambda\rho}g_{\mu\nu})$.

2. *2D Anti-de Sitter Spacetime*

Consider the 2D Anti-de Sitter spacetime with metric

$$ds^2 = a^2 \left(-\cosh^2\rho \, d\tau^2 + d\rho^2\right).$$

a) Calculate all the components of the affine connection $\Gamma^\mu_{\nu\lambda}$.

b) Calculate all the components of the Ricci tensor $R_{\mu\nu}$ and the curvature scalar R.

c) Show that $R_{\mu\nu\lambda\sigma} = \frac{1}{a^2}\left(g_{\mu\lambda}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\lambda}\right)$.

d) Suppose that the 2D Anti-de Sitter spacetime is the solution to Einstein's equations with some energy-momentum tensor $T_{\mu\nu}$. What is $T_{\mu\nu}$ in terms of a and $g_{\mu\nu}$?

e) Let $r = a\sinh\rho$ and $t = a\tau$. Write the metric in r, t coordinates.

3. *Divergence in Spherical Coordinates*

Consider spherical coordinates (r, θ, ϕ), which are related to Cartesian coordinates (x, y, z) by,

$$\begin{aligned} x &= r \sin\theta \cos\phi \\ y &= r \sin\theta \sin\phi \\ z &= r \cos\theta. \end{aligned}$$

a) If the components of a vector in Cartesian coordinates are V^x, V^y, V^z, then what are the components of that vector in spherical coordinates, V^r, V^θ, V^ϕ?

b) Using the covariant expression for the divergence,

$$D_\mu V^\mu = \frac{1}{\sqrt{g}} \partial_\mu \left(\sqrt{g} V^\mu \right),$$

calculate the divergence $\nabla \cdot \mathbf{V}$ in 3D Euclidean space in spherical coordinates.

4. *Rindler Space*

Consider the 2D spacetime whose metric is given by

$$ds^2 = d\rho^2 - \rho^2 \, d\eta^2.$$

a) Calculate the components of the curvature tensor $R_{\mu\nu\lambda\sigma}$.

b) What can you infer about this spacetime from your results of part (a)?