Physics 786, Spring 2014
Problem Set 4, Due Wednesday, March 12, 2014.

1. *Index Contraction*

a) If A^μ and B^ν are vectors under general coordinate transformations, then show that $A^\mu B_\mu = A^\mu B^\nu g_{\mu\nu}$ is a scalar.

b) Show that the covariant derivative of $A^\mu B_\mu$ is

$$D_\nu(A^\mu B_\mu) = \partial_\nu(A^\mu B_\mu).$$

2. *Covariant derivative of the metric*

a) Show that $g_{\mu\nu;\lambda} = 0$.

b) Show that $\delta_\mu{}^\nu{}_{;\lambda} = 0$.

3. *Harmonic Coordinates*

Show that the harmonic coordinate conditions $g^{\mu\nu}\Gamma^\lambda_{\mu\nu} = 0$ are equivalent to the conditions

$$\frac{\partial}{\partial x^\mu}\left(\sqrt{g}\, g^{\mu\lambda}\right) = 0.$$

4. *Basis Vectors and the Connection*

Consider the unit 2-sphere, parametrized by spherical coordinates θ, ϕ:

$$\mathbf{X} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta).$$

a) Construct the basis vectors on the tangent space, $\mathbf{e}_\theta$ and $\mathbf{e}_\phi$.

b) Calculate the components of the metric $g_{ij} = \mathbf{e}_i \cdot \mathbf{e}_j$ in spherical coordinates.

c) Calculate the components of the Christoffel symbols Γ^k_{ij} using

$$\mathbf{e}_l \cdot \partial_j \mathbf{e}_i = \Gamma^k_{ij}\, g_{kl}.$$