Physics 786, Spring 2014
Problem Set 3 Due Wednesday, February 19, 2014.

1. *Gravitational Waves*

a) Suppose that a gravitational plane wave has wavevector $k^\mu = (k, 0, 0, k)$ and polarization tensor $\epsilon_{\mu\nu}$ satisfying the harmonic gauge condition,

$$k^\mu \epsilon_{\mu\nu} = \tfrac{1}{2} k_\nu \epsilon_\mu{}^\mu .$$

Show that the components of $\epsilon_{\mu\nu}$ are related as follows:

$$\begin{aligned} \epsilon_{01} &= -\epsilon_{31}, \quad \epsilon_{02} = -\epsilon_{32}, \quad \epsilon_{22} = -\epsilon_{11}, \\ \epsilon_{03} &= -\tfrac{1}{2}\left(\epsilon_{33} + \epsilon_{00}\right). \end{aligned}$$

b) Show that by making a gauge transformation which preserves the harmonic gauge condition, $\epsilon_{\mu\nu}$ from part (a) can be put in the form,

$$\epsilon_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \epsilon_+ & \epsilon_\times & 0 \\ 0 & \epsilon_\times & -\epsilon_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

c) Consider a wavepacket $h_{\mu\nu} = f(z-t)\epsilon_{\mu\nu} + \text{c.c.}$, with $f(z-t)$ a positive wavefunction spread over some width L, and $\epsilon_{\mu\nu}$ as in part (b). Three particles in the $x^1 - x^2$ plane have spatial coordinates $\mathbf{x}_1 = (0,0,0)$, $\mathbf{x}_2 = (a, a, 0)$, and $\mathbf{x}_3 = (-a, a, 0)$. We may assume $a \ll L$.

Calculate the time evolution of the proper distance between pairs of particles 1 and 2, and between particles 1 and 3, as the gravitational wave passes. Compare the time evolution of these two proper distances.

d) Show that under a rotation of the coordinates by angle θ about the x^3-axis, the combination $\epsilon_+ \pm i\epsilon_\times$ transforms as,

$$\epsilon_+ \pm i\epsilon_\times \rightarrow e^{\pm 2i\theta}\left(\epsilon_+ \pm i\epsilon_\times\right).$$

Hence, gravitational waves have helicity ± 2.