Problems With the Old Standard Cosmological Model

1) The Horizon Problem

The universe has a finite age, so we are in causal contact with regions only a limited distance away. The surface which separates the region in causal contact from the region out of causal contact is called a <u>particle horizon</u>.

Suppose we are at $r=0$, and a light signal is emitted at r_e at coordinate time t_e. The signal is received at $r=0$ at time $t_r = t_0$ (today).

$ds^2=0$ along null trajectory $\Rightarrow dt^2 = R^2(t)\dfrac{dr^2}{1-kr^2}$

$$\int_0^{r_e} \frac{dr}{\sqrt{1-kr^2}} = \int_{t_e}^{t_0} \frac{dt}{R(t)}$$

As $t_e \to 0$, the particle horizon $r_H(t_0)$ is defined by

$$\boxed{\int_0^{r_H(t_0)} \frac{dr}{\sqrt{1-kr^2}} = \int_0^{t_0} \frac{dt}{R(t)}}$$

if the right-hand-side is finite.

Assuming the universe begins in a radiation dominated era, $\rho = \rho_0 \dfrac{R_0^4}{R^4}$.

Early universe:

$$\frac{R}{R_0} \ll 0 \rightarrow \dot{R}^2 = \frac{8\pi G}{3}\rho R^2 = \frac{8\pi G}{3}\rho_0 \frac{R_0^4}{R^2}$$

$$\frac{dR}{dt} = \sqrt{\frac{8\pi G}{3}\rho_0}\,\frac{R_0^2}{R}$$

$$\int_0^R R\,dR = \sqrt{\frac{8\pi G}{3}\rho_0}\,R_0^2 \int_0^t dt$$

$$\frac{R^2}{2} = \sqrt{\frac{8\pi G}{3}\rho_0}\,R_0^2\, t$$

$$R(t) = \left(\frac{24\pi G}{3}\rho_0\right)^{1/2} R_0\, t^{1/2} \sim t^{1/2}$$

Hence, $\int_0^{t_0} \frac{dt}{R(t)}$ is finite, so $\underline{\underline{r_H(t_0) \text{ is finite}}}$

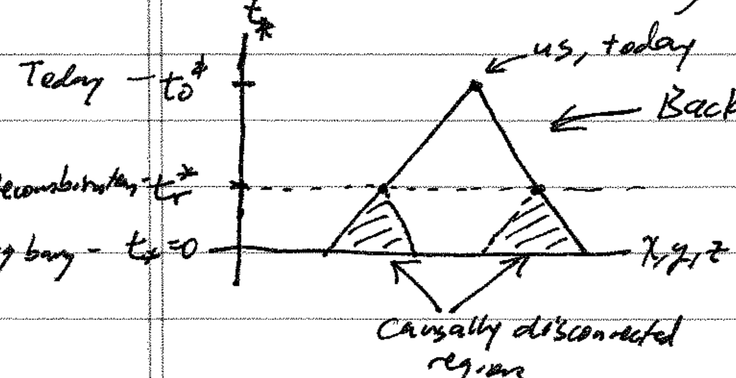
Assume $k=0$ (flat universe).

$$ds^2 = -dt^2 + R^2(t)\left(dx^2+dy^2+dz^2\right)$$

Define t_* by $\frac{dt}{R(t)} \equiv dt_* \Rightarrow t_*(t) = \int_0^t \frac{dt}{R(t)}$

$$ds^2 = -R^2(t)\,dt_*^2 + R^2(t)\left(dx^2+dy^2+dz^2\right)$$
$$= R^2(t)\left(-dt_*^2 + dx^2+dy^2+dz^2\right)$$

In these coordinates light rays move along 45° lines in ~~x~~ t.



Recombination! $t^r \sim 4\times10^5$ yrs
- Hydrogen atoms form,
long wavelength light sees neutral
atoms → radiation is frozen
⇒ 2.7°K $\underline{\text{Cosmic background radiation}}$

☆ <u>Puzzle</u>: Why is the Cosmic Microwave Background so uniform, even between what should be causally disconnected regions of spacetime?

2) Flatness Problem

Natural length scale in gravity: $l_{Planck} = \left(\frac{\hbar G}{c^3}\right)^{1/2} = 1.6 \times 10^{-33}$ cm

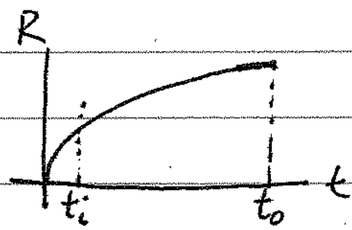
How did the universe get so big?

$$H^2 + \frac{K}{R^2} = \frac{8\pi G}{3}\rho \quad , \qquad \rho_c = \frac{3H^2}{8\pi G}$$

$$1 + \frac{K}{R^2 H^2} = \frac{8\pi G}{3}\frac{\rho}{H^2} = \frac{\rho}{\rho_c}$$

Density parameter: $\boxed{\Omega \equiv \frac{\rho}{\rho_c}}$

$\boxed{\Omega - 1 = \frac{K}{\dot{R}^2}}$. Observations indicate that today $\Omega \simeq 1$.



$\dot{R}$ was much larger in the past

$\rightarrow \Omega$ was much closer to 1 in the past

$\Omega(1\,\text{sec}) - 1 \lesssim 10^{-16}$.

☆: Puzzle: why was $\Omega \approx 1$ in the past?

3) Monopole Problem

Grand unified theories predict that magnetic monopoles would have been created in the universe, and should be plentiful today.

☆ Puzzle: Where are the magnetic monopoles?

The Cosmological Constant and Inflation

While trying to understand whether a static universe may be consistent with general relativity, Einstein introduced the cosmological constant into the Einstein Eqs.:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}$$

Λ ← cosmological constant

Recall that the form of the Einstein Eqs. was dictated by covariant conservation $D^\mu (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) = 0$. The metric is also covariantly conserved, $D^\mu g_{\mu\nu} = 0$, so the addition of the cosmological constant is consistent with the principles of general relativity.

The Friedmann Eqs are modified by Λ:

$$\boxed{\dot{R}^2 + K = \frac{8\pi G}{3} \rho R^2 + \frac{\Lambda}{3} R^2}$$

A vacuum energy would act as a cosmological constant:

$$T_{\mu\nu}^{vac} = -\rho_{vac}\, g_{\mu\nu}$$

$$\rightarrow \boxed{\Lambda = 8\pi G \rho_{vac}}$$

Compare with perfect fluid:

$$T_{\mu\nu} = p\, g_{\mu\nu} + (p+\rho) U_\mu U_\nu$$
$$= -\rho_{vac}\, g_{\mu\nu}$$

$\Big\}$ $\rho = \rho_{vac}$, $p = -\rho_{vac}$

Vacuum energy equation of state: $\boxed{p = -\rho}$

— Negative pressure fluid!

Consider a universe dominated by vacuum energy:

Friedmann eqn: $\ddot{R} = -\frac{4\pi G}{3}(\rho + 3p)R = \frac{8\pi G}{3}\rho_{vac} R$

Define $\chi^2 = \frac{8\pi G}{3}\rho_{vac} \rightarrow \ddot{R} = \chi^2 R$

$$\dot{R}^2 + K = \frac{8\pi G}{3}\rho R^2 = \chi^2 R^2$$

$$D_\nu T^{\mu\nu} = 0 \rightarrow \frac{d}{dR}(\rho_{vac} R^3) = -3 p_{vac} R^2 = 3\rho_{vac} R^2$$

$$3\rho_{vac} R^2 + R^3 \frac{d\rho_{vac}}{dR} = 3\rho_{vac} R^2$$

$$\rightarrow \frac{d\rho_{vac}}{dR} = 0 \longrightarrow \boxed{\rho_{vac} = \text{constant}}$$

$\Rightarrow \chi^2 = \text{const.}$

Large $\chi^2 R^2$ — ignore K.

Solution: $R \sim e^{\chi t} \iff$ <u>Inflation</u>

<u>Example</u>: Solution for all R with $K=1$:

$$R = \frac{1}{\chi}\cosh(\chi t) \sim \frac{e^{\chi t}}{2\chi} \text{ for large } t.$$

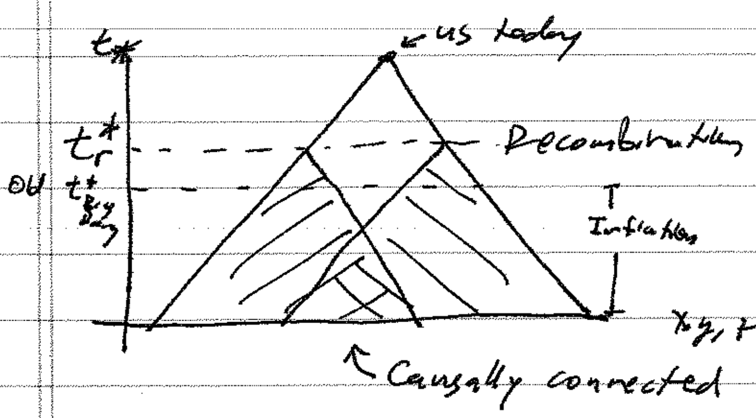
$$\dot{R} \sim \tfrac{1}{2} e^{\chi t}$$

$$\Omega - 1 = \frac{K}{\dot{R}^2} \sim (2\ e^{-\chi t})^2 \quad (K=1)$$

$\Omega \rightarrow 1$ exponentially quickly with χt.

— Explains flatness problem.

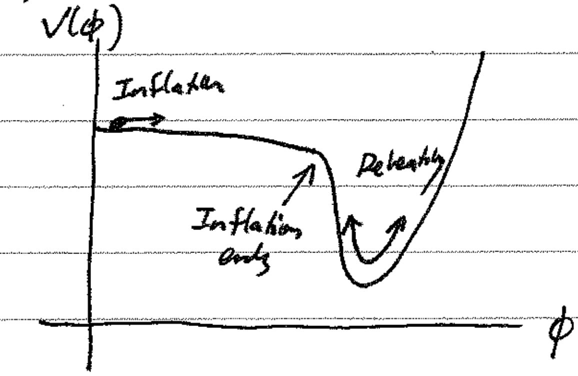
Universe expands exponentially quickly during inflation. Then stops inflating while the universe reheats, beginning the radiation domination era.



- Inflation explains the horizon problem. (if universe expands $> 10^{26}$ times)
- Inflation also explains the monopole problem
  - Density of monopoles decreases as the universe inflates.

Guth's Inflation: Vacuum energy from false vacuum (1980)
- End of inflation from bubble nucleation
  - difficult to reheat the universe w/ radiation

Linde; Albrecht, Steinhardt: Slow-Roll inflation = New Inflation (1983)



"Inflaton"

Scalar field ϕ slowly rolls down potential.

- Predicts spectrum of density fluctuations, Cosmic microwave background.

Today - There is strong evidence that the universe is currently undergoing a period of accelerated expansion. This could be due to a cosmological constant or some other fluid with $p = w\rho$, $w < -\frac{1}{3}$. Current bounds are roughly $w = -1.0 \pm 0.15$ assuming a constant w, consistent with the cosmological constant interpretation.

The most direct evidence for the accelerated expansion comes from Type Ia Supernova studies of the brightness vs. redshift curve.

1998 High-z Supernova Search team

1999 Supernova Cosmology Project

Cosmological Constant Problem(s):

1) The natural scale for the cosmological constant would seem to be $(M_{Planck})^4$, which is about 10^{120} times larger than observed. Why is it so small?

2) There is also a coincidence, that we happen to exist at exactly the right time between matter domination and vacuum energy domination, so that we have an interesting universe to observe, not devoid of other galaxy clusters.