**Physics 786, Spring 2012**
**Problem Set 7** Due Monday, April 23, 2012.


1. *Death by Black Hole*

Suppose a two-meter-tall human being falls feet-first into a black hole with the mass of the sun. Suppose the human can withstand the tidal acceleration gradient until the feet would accelerate 1000 m/s² more than than the head along a geodesic. What value of $r$ in standard coordinates do the feet reach before the human dies?

*Hint*: the tidal acceleration gradient is determined from the geodesic deviation $\frac{D^2}{D\tau^2}(\delta x^\mu)$.


2. *Encircling the Universe*

Show that a photon emitted at the big bang travels once around the universe in a matter-dominated $k = +1$ FRW universe (with vanishing cosmological constant).


3. *FRW Universe with Cosmological Constant*

a) Beginning with the FRW form of the metric and assuming the energy-momentum tensor of a perfect fluid, show that if Einstein's equations are modified to include a cosmological constant:

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R - \Lambda\, g_{\mu\nu} = -8\pi G_N T_{\mu\nu},$$

then the scale factor of the universe satisfies

$$\dot{R}^2 + k = \frac{8\pi G_N}{3}\rho R^2 + \frac{\Lambda}{3}R^2.$$


b) Show that if $\Lambda$ is large enough a $k = 1$ universe can expand forever.