

## Physics 786, Spring 2012

Problem Set 6 Due Wednesday, April 4, 2012.

*Final Paper Assignment - Due last day of class*

For your final paper, you should identify and describe an experiment or application which either makes use of results from general relativity or provides a test of general relativity. You should explain in detail those aspects of general relativity which are relevant. Some examples include Gravity Probe B, LIGO, LISA, GPS devices, and cosmological observatories like WMAP. As a guideline, aim for 5 double-spaced pages.

### 1. *Binary Star Systems*

Assume the two stars in a binary star system each have mass  $M=1.39$  solar masses, with orbital period  $T=7.75$  hrs, and semimajor axis with respect to one of the stars  $a = 2R=1.95\times 10^6$  km.

- a) Assuming circular orbits find the power radiated in gravitational radiation in Watts.
- b) In the same system, find the change in the orbital period after each complete orbit.
- c) Imagine instead that one of the stars has mass  $M_1=2.77$  solar masses but is treated as static while a much lighter star with mass  $M_2=0.14$  solar masses orbits in a circular orbit around it with radius  $R=1.95\times 10^6$  km. Find the power radiated in gravitational radiation in Watts. Compare your result with part (a).

### 2. *Killing Vectors*

- a) Consider 2D Euclidean space in Cartesian coordinates  $x, y$ . Find the Killing vectors related to translations and rotation about  $x = y = 0$  in these coordinates.
- b) What are the corresponding “constants of the motion” and their phys-

ical interpretation?

### 3. *Schwarzschild Spacetime*

a) A massive test particle is released from  $r = R > 2GM$  in the Schwarzschild geometry (in standard coordinates), and falls radially inward. Show that the following correctly parametrizes the trajectory:

$$r = \frac{R}{2}(1 + \cos \eta)$$
$$\tau = \frac{R}{2} \left( \frac{R}{2GM} \right)^{1/2} (\eta + \sin \eta).$$

b) Show that the proper time elapsed when the particle reaches  $r = 2GM$  is finite.

### 4. *More Schwarzschild*

A massive test particle is at  $r = r_0 < 2GM$  at  $t = 0$ .

a) Show that

$$\left| \frac{dr}{d\tau} \right| \geq \sqrt{\frac{2GM}{r} - 1}.$$

b) Show that the test particle necessarily reaches  $r = 0$  in finite proper time.