Problem Set 5 Due Wednesday, March 21, 2012.

1. *Curvature of the Two-Sphere*

Consider the two-sphere with metric

$$ds^2 = a^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right).$$

a) Calculate all the components of the affine connection $\Gamma^\mu_{\nu\lambda}$.

b) Calculate all the components of the Ricci tensor R_{ij} and the Gaussian curvature, $K = -R/2$, of the two-sphere.

Hint: In 2D, $R_{\lambda\mu\nu\rho} = \frac{1}{2}R(g_{\lambda\nu}g_{\mu\rho} - g_{\lambda\rho}g_{\mu\nu})$.

2. *2D de Sitter Spacetime*

Consider the 2D de Sitter spacetime with metric

$$ds^2 = a^2 \left(-du^2 + \cosh^2 u \, d\varphi^2\right).$$

a) Calculate all the components of the affine connection $\Gamma^\mu_{\nu\lambda}$.

b) Calculate all the components of the Ricci tensor $R_{\mu\nu}$ and the curvature scalar R.

3. *Covariant derivative of the metric*

Show that $g_{\mu\nu;\lambda} = 0$.

4. *Harmonic Coordinates*

Show that the harmonic coordinate conditions $g^{\mu\nu}\Gamma^\lambda_{\mu\nu} = 0$ are equivalent to the conditions

$$\frac{\partial}{\partial x^\mu}\left(\sqrt{g}\, g^{\mu\lambda}\right) = 0.$$