Physics 786, Spring 2012
Problem Set 4 Due Wednesday, March 14, 2012.

1. *Gravitational Radiation by Binary Star*

Suppose that a binary star system is composed of two stars of approximately equal mass M, separated by a distance $2r$, and rotating in a circular orbit about their center of mass nonrelativistically.

Star A follows the trajectory

$$x_A = r\ \cos(\omega t), \quad y_A = r\ \sin(\omega t), \quad z_A = 0.$$

Star B follows the trajectory

$$x_B = -r\ \cos(\omega t), \quad y_B = -r\ \sin(\omega t), \quad z_A = 0.$$

a) Using Newtonian mechanics, find ω in terms of G_N, M, and r.

b) The energy density of the nonrelativistic binary star system takes the form

$$T^{00} = M\ \left[\delta^3\left(\mathbf{x} - \mathbf{x}_A(t)\right) + \delta^3\left(\mathbf{x} - \mathbf{x}_B(t)\right)\right].$$

Calculate the quadrupole moments D^{ij}.

c) Calculate the spatial components of the gravitational radiation field $\overline{h}_{ij}$ in the radiation zone.

2. *Transformation of Covariant Derivative*

a) Show that under coordinate transformations, the covariant derivative $V^{\mu}_{\ ;\nu}$ transforms as a tensor.

b) Show that under coordinate transformations, the covariant derivative $V_{\mu;\nu}$ transforms as a tensor.

3. *Spherical Coordinates*

Spherical coordinates are defined in terms of the Cartesian coordinates x, y, z, by:

$$\begin{aligned} x &= r\,\sin\theta\,\cos\varphi \\ y &= r\,\sin\theta\,\sin\varphi \\ z &= r\,\cos\theta \end{aligned}$$

a) Show that the line element takes the form

$$ds^2 = dr^2 + r^2\,\left(d\theta^2 + \sin^2\theta\,d\varphi^2\right).$$

b) Find the volume element d^3x in spherical coordinates.

4. *Geodesics in Polar Coordinates*

Consider the 2D plane described in polar coordinates, with line element

$$ds^2 = dr^2 + r^2\,d\theta^2.$$

a) Calculate all of the components of the affine connection in these coordinates.

b) Show that any straight line satisfies the geodesic equation in these coordinates.

c) Find the volume element d^2x in polar coordinates.

d) Find the 2D Laplacian $\nabla^2 f$ in polar coordinates.