Physics 786, Spring 2012
Problem Set 2 Due Wednesday, February 15, 2012.

1. *Gravitational Redshift by the Sun*

Calculate the ratio of the frequency of light emitted at the surface of the sun to the frequency of that light as observed on Earth. The mass of the sun is 2.0×10^{30} kg and the radius of the sun is 0.70×10^{6} km. You may assume that the gravitational potential at Earth is negligible compared to that at the sun.

2. *Gravitational Twin Paradox*

Two cats are born at precisely the same time in the same litter, but are separated at birth. One lives in an apartment 10 meters above the other. After 15 years the cat on the top floor is brought downstairs to meet its sister. As a result of Earth's gravity, which cat is older and by how much?

3. *Coordinate Transformation of the Affine Connection*

Given the affine connection $\Gamma^{\lambda}_{\mu\nu}$ in a coordinate system x^{μ} calculate $\Gamma'^{\lambda}_{\mu\nu}$, the affine connection in the coordinate system x'^{μ}.

Hints: $\Gamma^{\lambda}_{\mu\nu}$ is not a tensor under general coordinate transformations. You might find it easiest to start with the definition of $\Gamma^{\lambda}_{\mu\nu}$ in Eq. (3.2.4) in Weinberg.

4. *The Angular Momentum Tensor*

Define
$$M^{\gamma\alpha\beta} \equiv x^{\alpha}T^{\beta\gamma} - x^{\beta}T^{\alpha\gamma},$$
where $T^{\beta\gamma}$ is the symmetric conserved energy-momentum tensor.

a) Show that $M^{\gamma\alpha\beta}$ is conserved, *i.e.*
$$\partial_{\gamma}M^{\gamma\alpha\beta} = 0.$$

b) What corresponding rank-2 tensor is time independent as a result of this conservation law?

c) Explain how the spatial components of the tensor found in part (b) are related to the angular momentum of the system.