Physics 786, Fall 2018
Problem Set 9, Due Monday, November 19.

1. *Final Paper*

You should already have a topic for your final paper. What is it?

2. *Killing Vectors*

a) Consider 2D Euclidean space in Cartesian coordinates x, y. Find the Killing vectors related to translations and rotation about $x = y = 0$ in these coordinates.

b) What are the corresponding "constants of the motion" along geodesics and their physical interpretation?

3. *Schwarzschild Trajectories*

a) A massive test particle is released from $r = R > 2GM$ in the Schwarzschild geometry (in standard coordinates), and falls radially inward. Show that the following correctly parametrizes the trajectory:

$$r = \frac{R}{2}(1 + \cos\eta)$$
$$\tau = \frac{R}{2}\left(\frac{R}{2GM}\right)^{1/2}(\eta + \sin\eta).$$

b) Show that the proper time elapsed when the particle reaches $r = 2GM$ is finite.

4. *The Photon Sphere*

a) Find the radius of circular orbits (defined by the value of r in standard Schwarzschild coordinates) in terms of the black hole mass. The collection of circular orbits is called the photon sphere.

b) In standard coordinates, what is $d\phi/dt$ in the circular orbit with $\theta = \pi/2$?

5. *Death by Black Hole*

Suppose a two-meter-tall human falls feet-first into a black hole with the mass of the sun. Suppose the human can withstand the tidal acceleration gradient until the feet would accelerate 100 $\mathrm{m/s^2}$ more than than the head along a geodesic. What value of r in standard coordinates do the feet reach before the human dies?

Hint: the tidal acceleration gradient is determined from the geodesic deviation $\frac{D^2}{D\tau^2}(\delta x^\mu)$.