Physics 786, Fall 2018
Problem Set 8, Due Wednesday, November 7.

1. *Newtonian Stars*

Consider static, spherically symmetric solutions to Einstein's equations for a fluid with density $\rho(r)$ and pressure $p(r)$, with metric of the form,

$$ds^2 = -e^{2\phi(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right).$$

Assume the fluid is nonrelativistic, and consider the nonrelativistic limit of Einstein's equations for this system. Assume $\phi(0) = \lambda(0) = 0$, and

$$\rho(r) = \rho_0\left(1 - \frac{r^2}{R^2}\right)$$

for $r \leq R$, and $\rho(r) = 0$ for $r > R$.

a) Find the spacetime metric for $r < R$ and $r > R$. The metric should be continuous across $r = R$.

b) Find the pressure $p(r)$ in the star such that $p(R) = 0$.

2. *The Planck Mass and Planck Length*

For a particle of mass M, quantum mechanics is important at distance scales of order the Compton wavelength of the particle. What is the value of M such that the reduced Compton wavelength $\lambda_C = \hbar/(Mc)$ is equal to the Schwarzschild radius of the particle? What is the corresponding value of λ_C? Express your results in SI units.

The Planck mass is defined by $M_{\rm Pl} = \sqrt{\hbar c/G}$. Compare the value of M that you found with the Planck mass.