Physics 786, Fall 2018
Problem Set 6, Due Wednesday, October 24.

1. *Curvature of the Two-Sphere*

Consider the two-sphere of radius a, with metric

$$ds^2 = a^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right).$$

a) (Re)calculate the Christoffel symbols of the two-sphere of radius a. How are those Christoffel symbols related to those of the two-sphere with radius 1?

b) Calculate all the components of the Ricci tensor $R_{\mu\nu}$ and the Gaussian curvature, $K = -R/2$, of the two-sphere.

c) In 2D, $R_{\lambda\mu\nu\rho} = \frac{1}{2}R(g_{\lambda\nu}g_{\mu\rho} - g_{\lambda\rho}g_{\mu\nu})$. Check that this relation is true for the two-sphere.

2. *Rindler Space*

Consider the 2D spacetime whose metric is given by

$$ds^2 = d\rho^2 - \rho^2 \, d\eta^2.$$

a) Calculate the components of the curvature tensor $R_{\mu\nu\lambda\sigma}$.

b) What can you infer about this spacetime from your results of part (a)?

3. *2D Anti-de Sitter Spacetime*

Consider the 2D Anti-de Sitter spacetime with metric

$$ds^2 = a^2\left(-\cosh^2\rho\, d\tau^2 + d\rho^2\right), \quad a = \text{const.}$$

a) Calculate all the components of the affine connection $\Gamma^{\mu}_{\nu\lambda}$.

b) Calculate all the components of the Ricci tensor $R_{\mu\nu}$ and the curvature scalar R.

c) Suppose that the 2D Anti-de Sitter spacetime is the solution to Einstein's equations with some energy-momentum tensor $T_{\mu\nu}$. What is $T_{\mu\nu}$ in terms of a and $g_{\mu\nu}$?

d) Let $r = a\sinh\rho$ and $t = a\tau$. Write the metric in r, t coordinates.

4. *Weyl tensor*

Show that the Weyl tensor, which in four dimensions is given by

$$C_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa} - \frac{1}{2}\left(g_{\lambda\nu}R_{\mu\kappa} - g_{\lambda\kappa}R_{\mu\nu} - g_{\mu\nu}R_{\lambda\kappa} + g_{\mu\kappa}R_{\lambda\nu}\right)$$
$$+\frac{1}{6}R\left(g_{\lambda\nu}g_{\mu\kappa} - g_{\lambda\kappa}g_{\mu\nu}\right),$$

is traceless, *i.e.* vanishes upon contraction of any pair of its indices.

5. *Counting components of curvature*

In D spacetime dimensions, how many independent components are there in:
a) the Riemann tensor?
b) the Ricci tensor?
c) the Weyl tensor (*i.e.* the traceless part of the Riemann tensor)?