1. **Curvature of the Two-Sphere**

Consider the two-sphere of radius $a$, with metric

$$ds^2 = a^2 \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right).$$

a) (Re)calculate the Christoffel symbols of the two-sphere of radius $a$. How are those Christoffel symbols related to those of the two-sphere with radius 1?

b) Calculate all the components of the Ricci tensor $R_{\mu\nu}$ and the Gaussian curvature, $K = -R/2$, of the two-sphere.

c) In 2D, $R_{\lambda\mu\nu\rho} = \frac{1}{2}R(g_{\lambda\nu}g_{\mu\rho} - g_{\lambda\rho}g_{\mu\nu})$. Check that this relation is true for the two-sphere.

2. **Rindler Space**

Consider the 2D spacetime whose metric is given by

$$ds^2 = d\rho^2 - \rho^2 \, d\eta^2.$$ 

a) Calculate the components of the curvature tensor $R_{\mu\nu\lambda\sigma}$.

b) What can you infer about this spacetime from your results of part (a)?
3. 2D Anti-de Sitter Spacetime

Consider the 2D Anti-de Sitter spacetime with metric

\[ ds^2 = a^2 \left( -\cosh^2 \rho \, d\tau^2 + d\rho^2 \right), \quad a = \text{const}. \]

a) Calculate all the components of the affine connection \( \Gamma^\mu_{\nu\lambda}. \)

b) Calculate all the components of the Ricci tensor \( R_{\mu\nu} \) and the curvature scalar \( R. \)

c) Suppose that the 2D Anti-de Sitter spacetime is the solution to Einstein’s equations with some energy-momentum tensor \( T_{\mu\nu}. \) What is \( T_{\mu\nu} \) in terms of \( a \) and \( g_{\mu\nu}. \)

d) Let \( r = a \sinh \rho \) and \( t = a\tau. \) Write the metric in \( r, t \) coordinates.

4. Weyl tensor

Show that the Weyl tensor, which in four dimensions is given by

\[ C_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa} - \frac{1}{2} \left( g_{\lambda\nu} R_{\mu\kappa} - g_{\lambda\kappa} R_{\mu\nu} - g_{\mu\nu} R_{\lambda\kappa} + g_{\mu\kappa} R_{\lambda\nu} \right) \]

\[ + \frac{1}{6} R \left( g_{\lambda\nu} g_{\mu\kappa} - g_{\lambda\kappa} g_{\mu\nu} \right), \]

is traceless, i.e. vanishes upon contraction of any pair of its indices.

5. Counting components of curvature

In \( D \) spacetime dimensions, how many independent components are there in:

a) the Riemann tensor?

b) the Ricci tensor?

c) the Weyl tensor (i.e. the traceless part of the Riemann tensor)?