

Physics 786, Fall 2018

Problem Set 5, Due Wednesday, October 10.

1. *Index Contraction*

a) If A^μ and B^ν are vectors under general coordinate transformations, then show that $A^\mu B_\mu = A^\mu B^\nu g_{\mu\nu}$ is a scalar.

b) Show that the covariant derivative of $A^\mu B_\mu$ is

$$D_\nu(A^\mu B_\mu) = \partial_\nu(A^\mu B_\mu).$$

2. *Covariant derivative of the metric*

a) Show that $g_{\mu\nu;\lambda} = 0$.

b) Show that $\delta_\mu^\nu{}_{;\lambda} = 0$.

3. *Harmonic Coordinates*

The harmonic coordinate condition is $g^{\mu\nu}\Gamma_{\mu\nu}^\lambda = 0$. Show that this condition is equivalent to

$$\frac{\partial}{\partial x^\mu} (\sqrt{g} g^{\mu\lambda}) = 0.$$

4. *Basis Vectors and the Connection*

Consider the unit 2-sphere, parametrized by spherical coordinates θ, ϕ :

$$\mathbf{X} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta).$$

a) Construct the basis vectors on the tangent space, \mathbf{e}_θ and \mathbf{e}_ϕ .

b) Calculate the components of the metric $g_{ij} = \mathbf{e}_i \cdot \mathbf{e}_j$ in spherical coordinates.

c) Calculate the components of the Christoffel symbols Γ_{ij}^k using

$$\mathbf{e}_i \cdot \partial_j \mathbf{e}_i = \Gamma_{ij}^k g_{kl}.$$