Physics 786, Fall 2018
Problem Set 2, Due in the second class after returning from the storm.

1. *Free fall*

Repeat and complete the derivation from class that along the trajectory of a freely falling particle,

$$\frac{d}{d\tau}\left(g_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\right) = 0.$$

2. *Gravitational Redshift*

Suppose a radio signal from a GPS satellite is sent to the ground. If the satellite is 20,200 km above Earth's surface, by what fraction is the frequency of the radio wave increased *due to gravity* compared to the emitted frequency when observed on the ground?

3. *Geodesics in Scalar Gravity*

Suppose the metric took the form $g_{\mu\nu} = \eta_{\mu\nu}\left(1 + 2\phi(\mathbf{x}, \mathbf{t})\right)$, where ϕ is the gravitational potential.

a) If $\phi = gz$, where g is the acceleration of gravity near the earth and z is the vertical displacement from the ground, what are the nonvanishing components of the Christoffel symbols $\Gamma^\mu_{\nu\lambda}$?

b) In the Newtonian approximation, use the geodesic equation to calculate the acceleration of a freely falling massive particle $\frac{d^2\mathbf{x}}{dt^2}$.

4. *External Forces*

In the presence of an electromagnetic field, the equation of motion for a charged particle is modified from the geodesic equation as follows:

$$\frac{d^2X^\lambda}{d\tau^2} + \Gamma^\lambda_{\mu\nu}\frac{dX^\mu}{d\tau}\frac{dX^\nu}{d\tau} = -\frac{e}{m}F^\lambda_{\ \nu}\frac{dX^\nu}{d\tau}.$$

For nonrelativistic motion in a weak, static gravitational field, write an approximate form for the time component and spatial components of this equation in terms of the gravitational potential ϕ and the electromagnetic fields $\mathbf{E}$ and $\mathbf{B}$.