Physics 786, Fall 2018
Problem Set 10, Due Monday, December 3.

You should be working on your paper. You should aim for 10 double-spaced pages, but I care about the content, not the page count. I expect the paper to look like something you might find on the arXiv, except that I do not expect you to present novel research. You must describe how general relativity is relevant to your topic, and equations are a plus. The paper is due the last day of class: Wednesday, December 5.

1. *FRW Universe with Cosmological Constant*

If Einstein's equations are modified to include a cosmological constant:

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R - \Lambda\, g_{\mu\nu} = -8\pi G_N T_{\mu\nu},$$

then the scale factor of the FRW universe satisfies

$$\dot{a}^2 + k = \frac{8\pi G_N}{3}\rho a^2 + \frac{\Lambda}{3}a^2.$$

Show that if the constant Λ is large enough and $\rho > 0$ (but not constant), a $k = +1$ universe can expand forever.

2. *Redshift vs. Distance*

Assume an expanding FRW universe with Hubble constant $H = \dot{a}/a$. Show that for nearby galaxies with proper distance from us r, and with $Hr \ll c$, the redshift factor is related to r by

$$z = \frac{\lambda_{\rm rec} - \lambda_{\rm emit}}{\lambda_{\rm emit}} \approx Hr/c.$$

3. *Gravitational Radiation from Binary Black Holes*

Suppose two black holes in a binary black hole system each have 30 solar masses, with orbital period beginning at $T = 0.01$s when the radius of the circular orbit about the center of mass is $R = 140$ km.

a) Treating the system in a Newtonian approximation, find the power radiated in gravitational radiation in Watts.

b) In the same system, estimate the change in the orbital period after one complete orbit.