Physics 786, Fall 2018
Problem Set 10, Due Monday, December 3.

You should be working on your paper. You should aim for 10 double-spaced pages, but I care about the content, not the page count. I expect the paper to look like something you might find on the arXiv, except that I do not expect you to present novel research. You must describe how general relativity is relevant to your topic, and equations are a plus. The paper is due the last day of class: Wednesday, December 5.

1. **FRW Universe with Cosmological Constant**

If Einstein’s equations are modified to include a cosmological constant:

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = -8\pi G_N T_{\mu\nu},
\]

then the scale factor of the FRW universe satisfies

\[
\dot{a}^2 + k = \frac{8\pi G_N}{3} \rho a^2 + \frac{\Lambda}{3} a^2.
\]

Show that if the constant \(\Lambda\) is large enough and \(\rho > 0\) (but not constant), a \(k = +1\) universe can expand forever.

2. **Redshift vs. Distance**

Assume an expanding FRW universe with Hubble constant \(H = \dot{a}/a\). Show that for nearby galaxies with proper distance from us \(r\), and with \(Hr \ll c\), the redshift factor is related to \(r\) by

\[
z = \frac{\lambda_{\text{rec}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}} \approx H r / c.
\]

3. **Gravitational Radiation from Binary Black Holes**

Suppose two black holes in a binary black hole system each have 30 solar masses, with orbital period beginning at \(T = 0.01\) s when the radius of the circular orbit about the center of mass is \(R = 140\) km.

a) Treating the system in a Newtonian approximation, find the power radiated in gravitational radiation in Watts.
b) In the same system, estimate the change in the orbital period after one complete orbit.