Problems With the Old Standard Cosmological Model

1) The Horizon Problem

The universe has a finite age, so we are in causal contact with regions only a limited distance away. The surface which separates the region in causal contact from the region out of causal contact is called a particle horizon.

Suppose we are at \( r = 0 \) and a light signal is emitted at \( r_e \) at coordinate time \( t_e \). The signal is received at \( r = 0 \) at time \( t_r = t_0 \) (today).

\[
\begin{align*}
\mathrm{d}s^2 = 0 \text{ along null trajectory } & \Rightarrow \mathrm{d}t^2 = R^2(t) \frac{\mathrm{d}r^2}{1 - kr^2} \\
& \Rightarrow \int_0^{r_e} \frac{\mathrm{d}r}{\sqrt{1 - kr^2}} = \int_{t_0}^{t_e} \frac{\mathrm{d}t}{R(t)}
\end{align*}
\]

As \( t_e \to 0 \), the particle horizon \( r_\text{p}(t_0) \) is defined by

\[
\begin{align*}
\int_0^{r_\text{p}(t_0)} \frac{\mathrm{d}r}{\sqrt{1 - kr^2}} &= \int_{t_0}^{t_e} \frac{\mathrm{d}t}{R(t)} \\
& \text{if the right-hand-side is finite.}
\end{align*}
\]

Assuming the universe begins in a radiation-dominated era, \( \rho = \rho_0 \frac{R_0^4}{R^4} \).

Early universe:

\[
\frac{R}{R_0} \ll 1 \Rightarrow \frac{r^2}{R^2} = \frac{8 \pi G \rho}{3} R_0^2 = \frac{8 \pi G \rho_0 R_0^4}{3 R^2}
\]
\[ \frac{dR}{dt} = \sqrt{\frac{8 \pi G}{3} \rho_0 \frac{R_0^2}{R}} \]

\[ \int_0^t R \, dR = \int_0^t \sqrt{\frac{8 \pi G}{3} \rho_0} R_0^2 \, t \]

\[ \frac{R^2}{2} = \int_0^t \sqrt{\frac{8 \pi G}{3} \rho_0} R_0^2 \, t \]

\[ R(t) = \left( \frac{24 \pi G}{3} \rho_0 \right)^{1/2} R_0 \, t^{1/2} \sim t^{1/2} \]

Hence, \( \int_0^t \frac{dt}{R(t)} \) is finite, so \( R_H(t) \) is finite.

Assume \( k = 0 \) (flat universe).

\[ \frac{ds^2}{-d\tau^2 + R^2(t) \left( dx^2 + dy^2 + dz^2 \right)} \]

Define \( \tau \) by \( \frac{dt}{R(t)} = d\tau \Rightarrow \tau(t) = \int_0^t \frac{dt}{R(t)} \)

\[ ds^2 = -R^2(t) d\tau^2 + R^2(t) \left( dx^2 + dy^2 + dz^2 \right) \]

In these coordinates, light rays move along \( 45^\circ \) lines in \( \tau \).

---

Diagram:

- Today \( \rightarrow \)
- Recombining \( \rightarrow \)
- By now \( \rightarrow 0 \)

Causally disconnected region

\( \theta \rightarrow 2.7^\circ K \) Cosmic Background Radiation

**Puzzle:** Why is the Cosmic Microwave Background so uniform, even between what should be causally disconnected regions of space-time?
2) **Flatness Problem**

Natural length scale in gravity: $l_{\text{Planck}} = \left(\frac{5.6 \times 10^{-33}}{c^3}\right)^{\frac{1}{2}} = 1.6 \times 10^{-33} \text{ cm}$

How did the universe get so big?

$$H^2 + \frac{k}{R^2} = \frac{8\pi G}{3} \rho , \quad \rho_c = \frac{3H^2}{8\pi G}$$

$$1 + \frac{k}{R^2H^2} = \frac{8\pi G}{3} \frac{\rho}{H^2} = \frac{\rho}{\rho_c}$$

**Density parameter:** $\Omega = \frac{\rho}{\rho_c}$

$\Omega - 1 = \frac{k}{R^2}$

Observations indicate that today $\Omega \approx 1$.

$R$ was much larger in the past.

$\Omega(1\text{ sec})^{-1} \approx 10^{-16}$.

**Puzzle:** Why was $\Omega \approx 1$ in the past?

3) **Monopole Problem**

Grand unified theories predict that magnetic monopoles would have been created in the universe, and should be plentiful today.

**Puzzle:** Where are the magnetic monopoles?
The Cosmological Constant and Inflation

While trying to understand whether a static universe may be consistent with general relativity, Einstein introduced the cosmological constant into the Einstein Eqs.:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}$$

\( \Lambda \) \text{ cosmological constant}

Recall that the form of the Einstein Eqs. was dictated by covariant conservation, \( D^\mu (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) = 0 \). The metric is also covariantly conserved, \( D^\mu g_{\mu\nu} = 0 \), so the addition of the cosmological constant is consistent with the principles of general relativity.

The Friedmann Eqs are modified by \( \Lambda \):

$$H^2 + k = \frac{8\pi G}{3} \rho R^2 + \frac{\Lambda}{3} R^2$$

A vacuum energy would act as a cosmological constant:

$$T_{\mu\nu}^{\text{vac}} = -\Lambda g_{\mu\nu}$$

$$\rightarrow \Lambda = 8\pi G \rho_{\text{vac}}$$

Compare with perfect fluid:

$$T_{\mu\nu} = \rho g_{\mu\nu} + (p + \rho) U_{\mu} U_{\nu}$$

$$\rightarrow \rho = \rho_{\text{vac}}$$

$$p = -\rho_{\text{vac}}$$

Vacuum energy equations of state:

$$p = -\rho$$

- Negative pressure fluid!
Consider a universe dominated by vacuum energy.

Friedmann eqn: \( \dot{R}^2 = \frac{8\pi G}{3} (\rho + 3p)R = \frac{8\pi G}{3} \rho_{\text{vac}} R \)

Define \( x^2 = \frac{8\pi G}{3} \rho_{\text{vac}} \) → \( \dot{x}^2 R \)

\[ R^2 + k = \frac{8\pi G}{3} \rho R^2 = x^2 R^2 \]

\[ D_v T^m = 0 \rightarrow \frac{d}{dR} (\rho R^2) = -3 \rho R^2 = 3 \rho_{\text{vac}} R^2 \]

\[ 3 \rho_{\text{vac}} R^2 + R^2 \frac{d\rho_{\text{vac}}}{dR} = 3 \rho_{\text{vac}} R^2 \]

\[ \rightarrow \frac{d\rho_{\text{vac}}}{dR} = 0 \rightarrow \rho_{\text{vac}} = \text{constant} \]

\[ \rightarrow x^2 = \text{Const.} \]

Large \( x^2 R^2 \) → ignore \( k \).

Solution: \( R \sim e^{xt} \) \( \leftrightarrow \) Inflation

Example: Solution for \( 4dR \) with \( k = 1 \):

\[ R = \frac{1}{x} \cosh (xt) \sim \frac{e^{xt}}{2x} \text{ for large } t. \]

\[ \dot{R} \sim \frac{1}{2} e^{xt} \]

\[ \Omega - 1 = \frac{k}{R^2} \sim (2 e^{-xt})^2 \text{ (k = 1)} \]

\[ \Omega \rightarrow 1 \text{ exponentially quickly with } xt. \]

→ Explains flatness problem.
Universe expands exponentially quickly during inflation, then stops inflation while the universe reheats, beginning the radiation domination era.

- Inflation explains the horizon problem. \(10^{26}\) times
- Inflation also explains the monopole problem
  - Density of monopoles decreases as the universe inflates.

Guth's Inflation: Vacuum energy from false vacuum (1980)
- End of inflation from stable nucleation
  - Difficult to reheat the universe w/ nucleation


Scalar field \(\phi\) slowly rolls down potential.
- Predicts spectrum of density fluctuations, cosmic microwave background
We do C handy

Field slowly rolls down hill while experiencing quantum fluctuations, until the field speeds up closer to minimum of $V(\phi)$.

The initial state can be inhomogeneous.


Quantum fluctuations allow parts of the universe to continue inflating while inflations end in some region.

Most of the universe (by volume) is still inflating.

This is the generic situation on new and chaotic inflation.

Puzzles:
1) How to make predictions for our local region of the universe if the potential is complicated with many local minima as in string theory? - The multiverse. (Steinhardt)
2) Considering the 2nd law of thermodynamics and that we are far from equilibrium today, how did such apparently fine-tuned initial conditions arise? (Penrose)
Today - There is strong evidence that the universe is currently undergoing a period of accelerated expansion. This could be due to a cosmological constant or some other fluid with \( p = \rho \), \( w < -\frac{1}{3} \). Current bounds are roughly \( w = -1 \pm 0.15 \) assuming a constant \( \Lambda \) consistent with the cosmological constant interpretation.

The most direct evidence for the accelerated expansion comes from Type Ia Supernova studies of the brightness vs. redshift curve.

1998: High-z Supernova Search Team
1999: Supernova Cosmology Project

Cosmological Constant Problem(s):

1) The natural scale for the cosmological constant would seem to be \( (M_{\text{Planck}})^4 \), which is about \( 10^{120} \) times larger than observed. Why is it so small?

2) There is also a coincidence, that we happen to exist at exactly the right time between matter domination and vacuum energy domination, so that we have an interesting universe to observe, not devoid of other galaxy clusters.