Physics 772, Spring 2009
Problem Set 7 Due Thursday, April 30.

1. *Running of the Standard Model Gauge Couplings*

a) Calculate the one-loop beta functions for the SU(3), SU(2) and U(1) gauge couplings in the Standard Model.

b) According to part (a), is the SU(2) sector of the Standard Model asymptotically free? Can you give an explanation for why the weak interactions do not lead to confinement?

c) Suppose in addition to the Standard Model particle content there were an additional multiplet of left-handed fermions in the $(2J+1)$-dimensional representation of $SU(2)_W$. How large can J be if SU(2) is to remain asymptotically free? (Note that, in general, such fermions would need to be added to the theory in pairs in order to avoid the Witten anomaly, but you should ignore that problem here.)

d) Rewrite the differential equation defining the β-function as an equation for $1/\alpha$, where $\alpha = g^2/(4\pi)$.

e) Solve the tree-level renormalization-group equation for $1/\alpha_1$, $1/\alpha_2$, and $1/\alpha_3$, where $\alpha_i = g_i^2/(4\pi)$ and g_i are the Standard Model gauge couplings. Plot $1/\alpha_i$ vs. $\log_{10}(\sqrt{-q^2}/\text{GeV})$.

Use the following experimental data:

$$\begin{aligned}
\alpha_{em}^{-1}(M_Z) &= 127.9 \\
\sin^2\theta_W(M_Z) &= 0.2312 \\
\alpha_3(M_Z) &= 0.1182
\end{aligned}$$

f) At what energy scale is $\alpha_2 = \alpha_3$?