Physics 772, Spring 2009
Problem Set 5 Due Tuesday, April 14.

1. *Unitarity bound on the Higgs mass*

Recall that the existence of the Z boson (or some other new physics beyond the charged current interactions) could have been inferred from the requirement of perturbative unitarity in $\nu\overline{\nu} \to W^+W^-$ scattering. Similarly, today we expect there to be a Higgs boson or other new physics at around the TeV scale because otherwise scattering of longitudinal gauge bosons would become nonunitary. In this problem you will calculate the bound on the Higgs mass from elastic scattering of longitudinal W bosons.

a) Draw all the Feynman diagrams in the Standard Model that contribute to $W^+W^- \to W^+W^-$ scattering at tree level in R_ξ gauge.

b) If the W bosons are all longitudinal (*i.e.* they have helicity 0) then the Feynman amplitude for large energies and Higgs mass compared to m_Z and m_W takes the form,

$$M(W_L^+W_L^- \to W_L^+W_L^-) = -\sqrt{2}G_F m_H^2 \left[\frac{s}{s-m_H^2} + \frac{t}{t-m_H^2}\right],$$

where s and t are the Mandelstam variables and m_H is the Higgs mass.

Show that the $J=0$ partial wave amplitude in the center of mass frame is

$$a_0 = -\frac{G_F m_H^2}{8\pi\sqrt{2}}\left[2 + \frac{m_H^2}{s-m_H^2} - \frac{m_H^2}{s}\log\left(1+\frac{s}{m_H^2}\right)\right].$$

c) Find the strongest upper bound on the Higgs mass from $J=0$ partial wave unitarity in the limit $s/m_H^2 \gg 1$.

d) Consider the opposite limit $m_H^2/s \gg 1$. At what center of mass energy would unitarity first break down in longitudinal W scattering?

e) Extra credit: Calculate the Feynman diagrams using the Feynman rules at the back of Cheng & Li and derive the expression given in part (b).

2. *The ρ parameter with different Higgs representations*

a) While you're thinking about the Higgs boson, recall that due to the custodial symmetry in the Standard Model Higgs potential, $m_W^2 \approx m_Z^2 \cos^2\theta_W$. Show that if the Higgs boson instead transformed in the $(2T+1)$-dimensional representation of $SU(2)_W$ and had hypercharge Y, then

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} \approx \frac{4T(T+1) - Y^2}{2Y^2}.$$

b) Show that the condition $\rho = 1$ implies

$$(2T+1)^2 - 3Y^2 = 1.$$

3. *Precision electroweak constraints*

Calculate an observable (that we have not calculated in class or for homework) from the table in Section 10 of the 2008 Review of Particle Physics and compare with the experimental value.

See page 24 of
http://pdg.lbl.gov/2008/reviews/rpp2008-rev-standard-model.pdf