Physics 772, Spring 2009
Problem Set 3 Due Tuesday, March 17.

1. *Adjoint scalars and the Coulomb branch*

Consider an SU(2) gauge theory with a set of three real scalar fields $\Phi = (\phi_1, \phi_2, \phi_3)$ transforming in the triplet representation of the gauge group.

a) What are the SU(2) generators in the triplet representation (in some basis)?

b) What is the Lagrangian for this theory? Assume the scalar field potential has the form
$$V(\Phi) = -\mu^2 \Phi^T \Phi + \lambda(\Phi^T \Phi)^2.$$

c) If $\mu^2 > 0$, find the values of Φ that minimize the potential.

d) Choose one of these minima as the vacuum expectation value of Φ. Expand the potential about that minimum, and identify the would-be Goldstone bosons.

e) Identify the unbroken gauge group, and show that the number of would-be Goldstone bosons equals the number of broken generators of the gauge group.

f) By expanding the Lagrangian about the vacuum, show that there are the same number of massive gauge fields in the spectrum of this theory as there are would-be Goldstone bosons.

2. *Physically equivalent vacua*

Consider a U(1) gauge theory with a complex scalar field. The scalar field potential has the form

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2, \quad \mu^2, \lambda > 0.$$

Show that the spectrum of scalar fields and gauge fields is independent of the choice of vacuum.