Physics 722, Spring 2019
Problem Set 5
Due Thursday, March 14.

In this problem set you will work through most of the calculation of the renormalized one-loop electron vertex function in QED, as was outlined in class. If you get stuck, use the books or ask me.

1. Using the Feynman rules for QED write out the one-loop contribution to the renormalized electron vertex function as an integral over the loop momentum.

2. Combine denominators using Feynman's trick and express the one-loop vertex function as an integral over the loop momentum and Feynman parameters. Indicate the integration region for the Feynman parameters.

3. Complete the square to make the integrand invariant under Lorentz transformations of the shifted loop momentum.

4. Due to the Lorentz symmetry, the following identities hold:

$$\int \frac{d^4k}{(2\pi)^4}\, k^\mu\, f(k^2) = 0,$$

$$\int \frac{d^4k}{(2\pi)^4}\, k^\mu k^\nu\, f(k^2) = \int \frac{d^4k}{(2\pi)^4}\, g^{\mu\nu}\, k^2\, f(k^2)/4.$$

Use these relations to simplify your expression for the one-loop vertex function.

5. Use dimensional regularization to regularize the integral over the shifted loop momentum.

6. Your result is probably not in the desired form,

$$\tilde{\Gamma}^\mu(p,p') = e\gamma^\mu\, F_1(q^2) + \frac{ie\,\sigma^{\mu\nu}q^\nu}{2m}\, F_2(q^2).$$

Manipulate the gamma matrices to put the integral in the desired form. Identify $F_1(q^2)$ and $F_2(q^2)$. This will probably be the bulk of your work in this problem set.

7. Is the one-loop contribution to $F_1(q^2)$ UV divergent? What about $F_2(q^2)$? Explain why the nondivergent part had to be that way, arguing based on renormalizability of QED.