Physics 722, Spring 2019
Problem Set 1, due Thursday, Jan 31.

1. *Born Approximation and the Yukawa Potential*

a) Consider the theory of a spinor field $\psi(x)$ coupled to a scalar field $\phi(x)$, with Lagrangian

$$\mathcal{L} = \overline{\psi}\left(i\partial\!\!\!/ - M\right)\psi + \frac{1}{2}\left(\partial_\mu\phi\right)^2 - \frac{m^2}{2}\phi^2 - g\,\overline{\psi}\psi\phi.$$

By comparing the nonrelativistic limit of the lowest-order Feynman amplitude for the process $\psi + \psi \to \psi + \psi$ with the scattering amplitude in nonrelativistic quantum mechanics, derive the potential $V(\mathbf{x})$ of the corresponding nonrelativistic theory of ψ particles.

b) Is the interaction attractive or repulsive?

2. *Renormalization of Spinor Fields*

a) In class we argued that in any Lorentz-invariant theory with a parity symmetry, the vacuum-to-one-particle matrix element of a Dirac spinor field operator $\psi(x)$ is the same as in the free theory up to an overall rescaling that depends on the interactions.

We demonstrated this explicitly for the case that the one-particle state is spin-up in the rest frame of the particle. Repeat the argument in the case that the one-particle state is spin-down, and explain why for a generic one-particle state the vacuum-to-one-particle matrix element is the same as in the free theory up to an overall rescaling.

b) In a parity non-conserving theory we argued that, for spin-up one-particle states in the rest frame, we could still define a renormalized field $\widetilde{\psi}(x)$ that may be expressed as a linear combination of the bare field $\psi(x)$ and $\gamma_5\psi(x)$, such that the vacuum-to-one-particle matrix element of the renormalized field is the same as in the free theory. Repeat the argument in the case that the one-particle state is spin-down.

3. Show that for a Lorentz-scalar field $\phi(x)$,

$$\langle 0|\phi(0)|\mathbf{k}\rangle = \langle 0|\phi(0)|\mathbf{0}\rangle,$$

where $|\mathbf{0}\rangle$ is a one-particle state with vanishing spatial momentum and $|0\rangle$ is the vacuum. Recall that we used this relation in the derivation that the physical mass defined by the mass-shell condition $\omega_{\mathbf{k}}^2 - \mathbf{k}^2 = m^2$ in the interacting theory is the same as the location of the pole in the renormalized scalar-field two-point function.

Hint: Think about the unitary operation that converts $\phi(x)$ to $\phi(\Lambda^{-1}x)$ for Lorentz-transformation Λ.