Physics 722, Spring 2016
Problem Set 5
Due Thursday, March 3.

1. *Rosenbluth Formula*

This is problem 6.1 in Peskin and Schroeder.

Assume the vertex function for the proton is of the form,

$$\tilde{\Gamma}^\mu(p', p) = \gamma^\mu F_1(q^2) + \frac{i\,\sigma^{\mu\nu}\,q_\nu}{2m} F_2(q^2),$$

where p, p' are the ingoing, outgoing proton momenta, and $q = p' - p$ is the ingoing photon momentum, and $\sigma^{\mu\nu} = i/2\,[\gamma^\mu, \gamma^\nu]$. (A factor of electric charge e is factored out of $\tilde{\Gamma}$ as for the electron.)

The form factors for strongly interacting particles like the proton are generally difficult to calculate, but they can be determined experimentally. Consider scattering of an energetic electron with energy $E \gg m_e$ from a proton initially at rest. To leading order in e the electron vertex function can be approximated by the tree-level interaction vertex, while the proton electromagnetic form factors $F_1(q^2)$ and $F_2(q^2)$ contain information about the strong interactions.

At leading order in $\alpha = e^2/4\pi$, show that the elastic scattering cross section takes the Rosenbluth form,

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2\left[\left(F_1^2 - \frac{q^2}{4m^2}F_2^2\right)\cos^2\frac{\theta}{2} - \frac{q^2}{2m^2}\left(F_1 + F_2\right)^2 \sin^2\frac{\theta}{2}\right]}{2E^2\left[1 + \frac{2E}{m}\sin^2\frac{\theta}{2}\right]\sin^4\frac{\theta}{2}},$$

where θ is the lab frame scattering angle and F_1 and F_2 are evaluated at the momentum transfer q^2 associated with elastic scattering at this angle.