

Physics 722, Spring 2016

Problem Set 3, due Thursday, February 25.

1. Gamma Matrices in Even Dimensions

In dimensional regularization we analytically continue in the number of spacetime dimensions. It is natural to consider what the gamma matrices look like in other than four (integer) dimensions.

In d spacetime dimensions, we want to find d matrices satisfying:

$$\begin{aligned}\gamma_0^\dagger &= \gamma_0, & \gamma_i^\dagger &= -\gamma_i, & i &= 1, \dots, d-1, \\ \{\gamma_\mu, \gamma_\nu\} &= 2g_{\mu\nu}, & \mu, \nu &= 0, \dots, d-1.\end{aligned}$$

It's easier to construct matrices satisfying,

$$\begin{aligned}\gamma_\mu^\dagger &= \gamma_\mu, & \mu &= 1, \dots, d, \\ \{\gamma_\mu, \gamma_\nu\} &= 2\delta_{\mu\nu}, & \mu, \nu &= 1, \dots, d.\end{aligned}$$

You can get the gamma matrices you want from these by letting $\gamma_d \rightarrow \gamma_0$, $\gamma_j \rightarrow i\gamma_j$.

a) Assume d is even. Define,

$$\begin{aligned}a_1 &= \frac{1}{2}(\gamma_1 + i\gamma_2), \\ a_2 &= \frac{1}{2}(\gamma_3 + i\gamma_4), \\ &\vdots \\ a_{d/2} &= \frac{1}{2}(\gamma_{d-1} + i\gamma_d),\end{aligned}$$

where the gamma matrices in these expressions satisfy the second set of conditions above.

Show that,

$$\begin{aligned}\{a_i, a_j\} &= \{a_i^\dagger, a_j^\dagger\} = 0, \\ \{a_i, a_j^\dagger\} &= \delta_{ij} \quad i, j = 1, \dots, d/2.\end{aligned}$$

This is the algebra of raising and lowering operators for $d/2$ independent two-level systems.

b) In two dimensions, construct a matrix representation for a and a^\dagger . What are γ_1 and γ_2 in that representation?

c) In d even dimensions, what is the dimensionality of your representation of the gamma matrices? Evaluate $\text{Tr } 1$ and $\text{Tr } \not{p}\not{p}$ in that representation.

d) Check that $\prod_i \gamma_i$ anticommutes with all of the γ_i . This is the analog of γ_5 in any even dimension.

Comment: In odd dimensions the first $d - 1$ gamma matrices can be constructed as above, and $\gamma_d = \pm\gamma_1\gamma_2\cdots\gamma_{d-1}$ completes the gamma matrix algebra. There are two independent representations of the gamma matrix algebra in odd dimensions, differing in the sign of γ_d . These representations are exchanged by parity, and both representations appear in a parity-conserving theory.

2. Confirm that the one-loop photon self-energy calculation that we performed in class with dimensional regularization (Lecture Notes 7) is not modified by use of the d -dimensional gamma-matrix relations rather than the 4-dimensional gamma-matrix relations.