Physics 722, Spring 2016
Problem Set 3, due Thursday, February 18.

1. *Derivative interactions revisited*

Consider the renormalized fermion two-point function $\langle 0|\mathrm{T}\left[\tilde{\psi}(x)\overline{\tilde{\psi}}(0)\right]|0\rangle$ in QED.

The counterterm for the fermion kinetic term is $\mathcal{L}_{\mathrm{CT}} \supset C\overline{\tilde{\psi}}(x)\, i\not{\partial}\, \tilde{\psi}(x)$.

Considering the term in the expansion of $\mathrm{T}\exp(-i \int H_I\, dt)$ to first order in C as it appears in a contribution to the fermion two-point function, and neglecting the issue that the time derivative does not commute with the time-ordering operation, calculate from first principles the contribution of order C to the fermion self energy.

Use your result to confirm the Feynman rule for the counterterm C stated in class.

2. *Fermion self energy*

Consider the theory of a fermion $\psi(x)$ with mass m, Yukawa coupled to a real scalar field $\phi(x)$ with mass μ:

$$\mathcal{L} = \overline{\psi}(i\not{\partial} - m)\psi + \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\mu^2}{2}\phi^2 - g\overline{\psi}\psi\phi - \frac{\lambda_3}{3!}\phi^3 - \frac{\lambda_4}{4!}\phi^4 + \mathcal{L}_{\mathrm{CT}}.$$

a) Calculate the one-loop renormalized nucleon self energy $\widetilde{\Sigma}(\not{p})$. The renormalized self energy should satisfy $\widetilde{\Sigma}(m) = 0$ and $d\widetilde{\Sigma}/d\not{p}\,|_{\not{p}=m} = 0$. Use a hard momentum cutoff to regularize any divergent integrals appearing at intermediate stages of the calculation, and check that those divergences are cancelled in the renormalization procedure. Your result should be left in terms of integral(s) over a single Feynman parameter.

b)Does $\widetilde{\Sigma}(\not{p})$ have a branch cut? If so, what is the physical interpretation of the value of p^2 at the branch point (not at infinity)?

3. *Volume of a d-dimensional sphere*

a) Compute the $(d+1)$-dimensional integral

$$I_{d+1} \equiv \int d^{d+1}x\, e^{-(x_1^2+x_2^2+\cdots+x_{d+1}^2)}.$$

b) Write the integral in spherical coordinates and separate out the angular part of the integral. By a suitable change of variables, use the definition of the gamma function,

$$\Gamma(t) = \int_0^\infty x^{t-1} e^{-x}\, dx,$$

to evaluate the integral in terms of the gamma function and the volume of a d-dimensional unit sphere.

c) By comparing the results of parts (a) and (b), determine the volume of the d-dimensional unit sphere.