

Electroweak Interactions - The Standard Model

The electroweak interactions are governed by an $SU(2) \times U(1)$ gauge theory. The Higgs mechanism spontaneously breaks the gauge group to a $U(1)$ subgroup. The corresponding massless gauge field is identified w/ the photon, and mediates the electromagnetic interaction. The three massive gauge bosons are called the W^\pm and the Z .

Note: The $U(1)$ factor in $SU(2) \times U(1)$ describes hypercharge, and is not the same as the unbroken $U(1)$ describing electromagnetism. We will write $SU(2) \times U(1)_Y$ and $U(1)_{em}$ to distinguish the $U(1)$'s.

In the Standard Model there is a complex doublet of scalar fields with hypercharge $Y(\Phi) = 1$.

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \phi^+, \phi^0 \text{ complex scalar fields.}$$

Call the $SU(2)$ gauge bosons A_m^a , gauge coupling g
 $U(1)_Y$ gauge boson B_m , gauge coupling $\frac{g'}{2}$.

$$D_m \Phi = \left(\partial_m - i \frac{g}{2} \sigma^a A_m^a - i \frac{g'}{2} B_m \right) \Phi$$

$$\mathcal{L} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi)$$

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \quad \lambda, \mu > 0.$$

$$\text{Choose } \langle \Phi \rangle_0 \equiv \langle 0 | \Phi | 0 \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad v = \left(\frac{\mu^2}{\lambda} \right)^{1/2}$$

$$\text{Write } \Phi(x) = U^{-1}(\xi^a(x)) \begin{pmatrix} 0 \\ \frac{v + \eta(x)}{\sqrt{2}} \end{pmatrix}$$

$$\text{where } U(\xi^a) = \exp\left(i \xi^a \frac{\sigma^a}{2}\right).$$

Transform to Unitary gauge:

$$\begin{cases} \Phi'(x) = U(\xi^a(x)) \Phi(x) = \begin{pmatrix} 0 \\ \frac{v + \eta(x)}{\sqrt{2}} \end{pmatrix}, & \boxed{\eta(x) = \text{Higgs field}} \\ A'_\mu{}^a \frac{\sigma^a}{2} = U(\xi^a(x)) A_\mu^a \frac{\sigma^a}{2} U^{-1}(\xi^a(x)) + \frac{i}{g} U \partial_\mu U^{-1} \end{cases}$$

$$\begin{aligned} D_\mu \Phi' &= U D_\mu \Phi = U \left(\partial_\mu - \frac{ig}{2} \sigma^a A_\mu^a - \frac{ig'}{2} B_\mu \right) \Phi \\ &\stackrel{U \uparrow U}{=} \left(\partial_\mu - \frac{ig}{2} \sigma^a A'_\mu{}^a - \frac{ig'}{2} B_\mu \right) \Phi' \end{aligned}$$

The scalar field potential is:

$$V(\Phi') = -\frac{\mu^2}{2} (v + \eta(x))^2 + \frac{\lambda}{4} (v + \eta(x))^4$$

$$= \text{const.} + \left(-\frac{\mu^2}{2} \eta^2 + \frac{\lambda}{2} v^2 \eta^2 \right) + \lambda v \eta^3 + \frac{\lambda}{4} \eta^4$$

$$= \text{const.} + \mu^2 \eta^2 + \lambda v \eta^3 + \frac{\lambda}{4} \eta^4$$

$$\boxed{m_\eta = \sqrt{2} \mu} \quad \text{Higgs mass}$$

Gauge Field Mass Terms:

$$\mathcal{L} = \frac{1}{2} (0, v) \left(\frac{g}{2} \sigma^a A'_m{}^a + \frac{g'}{2} B_m \right) \left(\frac{g}{2} \sigma^b A'^{mb} + \frac{g'}{2} B^m \right) \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{v^2}{8} g^2 \left((A'_m{}^1)^2 + (A'_m{}^2)^2 \right) + \frac{v^2}{8} (g A'_m{}^3 - g' B_m)^2$$

$$\equiv M_W^2 W_m^+ W^{-m} + \frac{1}{2} M_Z^2 Z_m Z^m$$

where $W_m^\pm = (A'_m{}^1 \mp A'_m{}^2) / \sqrt{2}$, $M_W^2 = \frac{g^2 v^2}{4}$

$$\frac{1}{2} M_Z^2 Z_m Z^m = \frac{v^2}{8} (g A'_m{}^3 - g' B_m)^2$$

$$= \frac{v^2}{8} (g^2 + g'^2) \left(\frac{g A'_m{}^3 - g' B_m}{\sqrt{g^2 + g'^2}} \right)^2$$

$$\Rightarrow M_Z^2 = \frac{v^2}{4} (g^2 + g'^2)$$

↑ For canonically normalized kinetic term.

★ Define $\cos \theta_w = \frac{g}{\sqrt{g^2 + g'^2}}$, $\sin \theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}$

$$Z_m = \cos \theta_w A'_m{}^3 - \sin \theta_w B_m$$

↑ weak mixing angle, sometimes called the Weinberg angle.

Orthogonal combination: $A_m = \sin \theta_w A'_m{}^3 + \cos \theta_w B_m$

The normalization is chosen so that

$$\begin{aligned} & (\partial_m A'_\nu{}^3 - \partial_\nu A'_m{}^3)^2 + (\partial_m B_\nu - \partial_\nu B_m)^2 \\ &= (\partial_m Z_\nu - \partial_\nu Z_m)^2 + (\partial_m A_\nu - \partial_\nu A_m)^2 \end{aligned}$$

★ A_m has no mass term \rightarrow Photon

The weak mixing angle will appear in couplings to fermions \rightarrow measured in scattering experiments

Tree-level Standard Model prediction:

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1$$

The ρ parameter.

Custodial Symmetry in the Higgs Sector

The Higgs potential is of the form $V(\Phi) = V(\Phi^\dagger \Phi)$.

In terms of the real and imaginary parts of Φ ,

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} (\phi_1^+ + i\phi_2^+)/\sqrt{2} \\ (\phi_1^0 + i\phi_2^0)/\sqrt{2} \end{pmatrix}$$

$$\Phi^\dagger \Phi = \frac{1}{2} \left((\phi_1^+)^2 + (\phi_2^+)^2 + (\phi_1^0)^2 + (\phi_2^0)^2 \right)$$

Hence, the potential has an $O(4) \sim SU(2) \times SU(2)$ global symmetry.

The Higgs VEV breaks $O(4) \rightarrow O(3) \sim SU(2) =$ custodial symmetry

The $SU(2)$ gauge fields A_m^a , $a=1,2,3$ are a degenerate triplet under the unbroken $O(3)$, i.e. the mass terms have the form

$$\mathcal{L} \supset \frac{1}{2} M_W^2 \left[(A_m^1)^2 + (A_m^2)^2 + (A_m^3)^2 \right]$$

The trace of the (A_m^a, B) mass matrix $= \frac{M_Z^2}{2} + \frac{M_W^2}{2} = \frac{M_Z^2}{2}$

The determinant is $\frac{M_Z^2}{2} \cdot \frac{M_W^2}{2} = 0$. This determines the mass matrix:

$$\mathcal{L} \supset \frac{1}{2} (A_m^a, B_m) \begin{pmatrix} M_W^2 & M_W (M_Z^2 - M_W^2)^{1/2} \\ M_W (M_Z^2 - M_W^2)^{1/2} & M_Z^2 - M_W^2 \end{pmatrix} \begin{pmatrix} A_m^3 \\ B_m \end{pmatrix}$$

Diagonalizing the mass matrix, it immediately follows that

$$\cos \theta_w = \frac{M_w^2}{(M_w^4 + M_w^2(M_Z^2 - M_w^2))^{1/2}} = \frac{M_w}{M_Z}$$

Hence, the prediction $\rho = 1$ is a consequence of the custodial symmetry in the Higgs sector.

Fermion Gauge Couplings

Consider a single generation of fermions:

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad Y(Q_L) = 1/3 \quad \leftarrow \text{Define } Q_L' = U(S^a) Q_L$$

$$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad Y(L_L) = -1 \quad \leftarrow \text{Define } L_L' = U(S^a) L_L$$

$$u_R, \quad Y(u_R) = 2/3$$

$$d_R, \quad Y(d_R) = -1/3$$

$$e_R, \quad Y(e_R) = -2$$

$$\left(\begin{array}{l} \nu_R \\ \nu_R \end{array} \right), \quad Y(\nu_R) = 0$$

↑
Hypercharges

$$\mathcal{L} = \bar{L}_L' \left(\frac{g}{2} \sigma^a A'^a - \frac{g'}{2} B \right) L_L' + \bar{Q}_L' \left(\frac{g}{2} \sigma^a A'^a + \frac{g'}{6} B \right) Q_L'$$

$$- \bar{e}_R g' B e_R + \bar{u}_R \frac{2g'}{3} B u_R - \bar{d}_R \frac{g'}{3} B d_R$$

Charged Current:
$$\mathcal{L}_{CC} = \bar{L}_L' \left(\frac{g}{2} \sigma^1 A'^1 + \frac{g}{2} \sigma^2 A'^2 \right) L_L' + \bar{Q}_L' \left(\frac{g}{2} \sigma^1 A'^1 + \frac{g}{2} \sigma^2 A'^2 \right) Q_L'$$

$$L_{CC} = \frac{g}{2} (\bar{\nu}_L', \bar{e}_L') \begin{pmatrix} 0 & A' - iA'^2 \\ A' + iA'^2 & 0 \end{pmatrix} \begin{pmatrix} \nu_L' \\ e_L' \end{pmatrix}$$

$$+ \frac{g}{2} (\bar{u}_L', \bar{d}_L') \begin{pmatrix} 0 & A' - iA'^2 \\ A' + iA'^2 & 0 \end{pmatrix} \begin{pmatrix} u_L' \\ d_L' \end{pmatrix}$$

$$= \frac{g}{\sqrt{2}} (\bar{\nu}_L' \gamma^m e_L' + \bar{u}_L' \gamma^m d_L') W_m^+$$

$$+ \frac{g}{\sqrt{2}} (\bar{e}_L' \gamma^m \nu_L' + \bar{d}_L' \gamma^m u_L') W_m^-$$

$$\equiv \frac{g}{\sqrt{2}} (J^{+\mu} W_\mu^+ + J^{-\mu} W_\mu^-), \text{ where}$$

$$\boxed{\begin{aligned} J^{+\mu} &= \bar{\nu}_L' \gamma^\mu e_L' + \bar{u}_L' \gamma^\mu d_L' \\ J^{-\mu} &= \bar{e}_L' \gamma^\mu \nu_L' + \bar{d}_L' \gamma^\mu u_L' \end{aligned}}$$

Neutral Currents

$$L_{NC} = \bar{L}_L' \left(\frac{g}{2} \sigma^3 A'^3 - \frac{g'}{2} B \right) L_L' + \bar{Q}_L' \left(\frac{g}{2} \sigma^3 A'^3 + \frac{g'}{6} B \right) Q_L' \\ - \bar{e}_R g' B e_R + \bar{u}_R \frac{2g'}{3} B u_R - \bar{d}_R \frac{g'}{3} B d_R$$

$$= \bar{\nu}_L' \left(\frac{g}{2} A'^3 - \frac{g'}{2} B \right) \nu_L' - \bar{e}_L' \left(\frac{g}{2} A'^3 + \frac{g'}{2} B \right) e_L'$$

$$+ \bar{u}_L' \left(\frac{g}{2} A'^3 + \frac{g'}{6} B \right) u_L' + \bar{d}_L' \left(-\frac{g}{2} A'^3 + \frac{g'}{6} B \right) d_L'$$

$$- \bar{e}_R g' B e_R + \bar{u}_R \frac{2g'}{3} B u_R - \bar{d}_R \frac{g'}{3} B d_R$$

L_{NC} can be written $L_{NC} = \sum_{\substack{\text{chiral} \\ \text{fermions} \\ f}} \bar{f} (g T^3 A^3 + \frac{g'}{2} Y(f) B) f$

where $T^3 = \frac{\sigma^3}{2}$ for the left-handed doublets

$T^3 = 0$ for the right-handed singlets

$Y(f)$ = hypercharge of fermion f .

Recall
$$\begin{pmatrix} Z_m \\ A_m \end{pmatrix} = \begin{pmatrix} \cos \theta_w & -\sin \theta_w \\ \sin \theta_w & \cos \theta_w \end{pmatrix} \begin{pmatrix} A_m^3 \\ B_m \end{pmatrix}$$

Hence,

$$\begin{pmatrix} A_m^3 \\ B_m \end{pmatrix} = \begin{pmatrix} \cos \theta_w & \sin \theta_w \\ -\sin \theta_w & \cos \theta_w \end{pmatrix} \begin{pmatrix} Z_m \\ A_m \end{pmatrix}$$

$$L_{NC} = \sum_f \bar{f} \left(g T^3 (\cos \theta_w \bar{Z} + \sin \theta_w A) + \frac{g'}{2} Y(f) (-\sin \theta_w \bar{Z} + \cos \theta_w A) \right) f$$

$$= \sum_f \left[\bar{f} \left(g T^3 \cos \theta_w - \frac{g'}{2} Y(f) \sin \theta_w \right) \bar{Z} f \right.$$

$$\left. + \bar{f} \left(g T^3 \sin \theta_w + \frac{g'}{2} Y(f) \cos \theta_w \right) A f \right]$$

Recall $\cos \theta_w = \frac{g}{(g^2 + g'^2)^{1/2}}$, $\sin \theta_w = \frac{g'}{(g^2 + g'^2)^{1/2}}$

Define $e = g \sin \theta_w = g' \cos \theta_w$

The electromagnetic interaction takes the form

$$L_{em} = \sum_f e \bar{f} (T^3 + \frac{1}{2} Y(f)) \gamma^m f A_m$$

$$\equiv e J_{em}^m A_m,$$

$$J_{em}^m = \sum_f \bar{f} (T^3 + \frac{1}{2} Y(f)) \gamma^m f$$

We therefore identify the electric charge

$$Q = T^3 + \frac{1}{2} Y$$

Fermion	T^3	Y	$Q = T^3 + Y/2$
u_L	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{2}{3}$
d_L	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{3}$
ν_L	$\frac{1}{2}$	-1	0
e_L	$-\frac{1}{2}$	-1	-1
u_R	0	$\frac{4}{3}$	$\frac{2}{3}$
d_R	0	$-\frac{2}{3}$	$-\frac{1}{3}$
e_R	0	-2	-1
(ν_R)	0	0	0

Z Couplings

$$\mathcal{L}_Z = \sum_f \bar{f} \left(g T^3 \cos \theta_w - \frac{g'}{2} Y(f) \sin \theta_w \right) \gamma^\mu f Z_\mu$$

$$= \sum_f \bar{f} \left[\frac{g}{\cos \theta_w} (1 - \sin^2 \theta_w) T^3 - \frac{g' \sin^2 \theta_w}{\cos \theta_w} \frac{Y(f)}{2} \right] \gamma^\mu f Z_\mu$$

↑ using $g' \cos \theta_w = g \sin \theta_w$

$$= \sum_f \bar{f} \frac{g}{\cos \theta_w} \left[T^3 - \sin^2 \theta_w Q(f) \right] \gamma^\mu f Z_\mu$$

$$= \frac{g}{\cos \theta_w} J_0^\mu Z_\mu, \text{ where } J_0^\mu = \sum_f \left[g_L^f \bar{f}_L \gamma^\mu f_L + g_R^f \bar{f}_R \gamma^\mu f_R \right]$$

$$g_{L,R}^f = T^3(f_{L,R}) - \sin^2 \theta_w Q(f)$$

Weak neutral-current couplings.

<u>Fermion</u>	<u>T^3</u>	<u>Q</u>	<u>g^F</u>
u_L	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{2} - \frac{2}{3} \sin^2 \theta_w$
d_L	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{2} + \frac{1}{3} \sin^2 \theta_w$
ν_L	$\frac{1}{2}$	0	$\frac{1}{2}$
e_L	$-\frac{1}{2}$	-1	$-\frac{1}{2} + \sin^2 \theta_w$
u_R	0	$\frac{2}{3}$	$-\frac{2}{3} \sin^2 \theta_w$
d_R	0	$-\frac{1}{3}$	$\frac{1}{3} \sin^2 \theta_w$
e_R	0	-1	$\sin^2 \theta_w$
(ν_R)	0	0	0