Summary:

The Feynman rules for the propagators are:

$$\underset{\phi_1'}{\overset{k\rightarrow}{\text{———}}} \qquad \frac{i}{k^2-2\mu^2+i\epsilon}$$

$$\underset{\phi_2'}{\overset{k\rightarrow}{\text{———}}} \qquad \frac{i}{k^2-\xi m^2+i\epsilon}$$

$$\mu \sim\sim\sim\sim \nu \qquad \frac{-i}{k^2-M^2+i\epsilon}\left[g^{\mu\nu}-(1-\xi)\frac{k^\mu k^\nu}{k^2-\xi m^2}\right]$$

$$= -i\left[\frac{g^{\mu\nu}-k^\mu k^\nu/m^2}{k^2-m^2+i\epsilon}+\frac{k^\mu k^\nu/m^2}{k^2-\xi m^2+i\epsilon}\right]$$

In the R_ξ gauge the would-be Goldstone boson ϕ_2' propagates, but the propagator depends on the gauge fixing parameter ξ.

But now the vector boson propagator falls like $\frac{1}{k^2}$ at large momentum, so the high energy behavior is manifestly more mild than in unitary gauge.

(Fujikawa, Lee, Sanda (1972); Yao (1973))

In QED in covariant gauges the Fadeev-Popov determinant is independent of the fields and can be factored out of the functional integral. However, in R_ξ gauge it's not so simple.

The gauge fixing condition is

$$G(A_\mu, \phi) = \partial_\mu A^\mu + \xi m \phi_2 - f(x) = 0$$

In terms of the shifted fields $\phi_1' = \phi_1 - v$, $\phi_2' = \phi_2$, the gauge invariance is (infinitessimally)

$$\phi_1' \to \phi_1'(x) - \alpha(x)\phi_2'(x)$$
$$\phi_2' \to \phi_2' + \alpha(x)(v + \phi_1')$$
$$A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \alpha(x)$$

The Fadeev-Popov determinant is

$$\det\left(\frac{\delta G}{\delta \alpha}\right) = \det\left(-\frac{1}{e}\partial_\mu\partial^\mu + \xi m(v + \phi_1')\right)$$

Introducing Fadeev-Popov ghost fields $c(x), \bar{c}(x)$, we can absorb the determinant into a functional integral over the ghosts, with ghost Lagrangian

$$\boxed{\mathcal{L}^{R_\xi}_{ghost} = \bar{c}\left(-\partial_\mu\partial^\mu + \xi m^2\left(1 + \frac{\phi_1'}{v}\right)\right)c}$$

ghost coupling to physical scalar.

The non-Abelian generalization of the Higgs mechanism is straightforward, as is the discussion of unitary gauge and R_ξ-gauge. (See Peskin + Schroeder Ch. 20.1 and 21.1, Cheng + Li Ch. 8.3, 9.2)

In the Standard Model, the <u>Higgs doublet</u> is an $SU(2)_W$ doublet and is charged under $U(1)_Y$.

weak gauge invariance — Hypercharge gauge invariance

The Higgs field acquires a vacuum expectation value, which spontaneously breaks $SU(2)_W \times U(1)_Y$ to $U(1)_{EM}$.

Electromagnetism

That is why, if the Standard Model is correct, the $W^\pm$ and Z bosons are heavy, while the photon is massless.