Physics 722, Spring 2008
Problem Set 8: Gauge Theory, Functional Integrals
Due Thursday, April 24.

1. *The Standard Model*

The Standard Model is an SU(3)×SU(2)×U(1) gauge theory. Consider one generation of quarks. The left-handed (chirality) quarks are in the fundamental representation of SU(3), the fundamental of SU(2), and have U(1) charge 1/6. The right-handed quarks are also in the fundamental of SU(3), but are SU(2) singlets. One SU(3) triplet of right-handed quark has U(1) charge 2/3, and another SU(3) triplet has U(1) charge -1/3.

a) What is the Lagrangian for the quarks and gauge fields, including the kinetic terms and gauge couplings? Be sure to clearly define all notation.

b) Are there any gauge-invariant mass terms? If not, explain.

c) Draw and evaluate the one-loop Feynman diagram(s) that contribute to the U(1) gauge-field self energy, regularizing in the $\overline{\rm MS}$ scheme. Express your result in terms of the photon self-energy in QED, perhaps in the limit of vanishing electron mass.

d) Draw the one-loop Feynman diagrams that contribute to the SU(3) gauge boson (gluon) self-energy.

e) Evaluate the quark loops in part (d).

Comment: In the full Standard Model, Yukawa couplings involving the Higgs field give rise to quark masses. You are not supposed to include those couplings for this problem.

2. *Functional Integral Quantization*

Using the functional integral for a free complex scalar field ϕ with mass m, evaluate the following correlation functions:

$$\langle 0|\phi(x)|0\rangle,\ \langle 0|T\left[\phi(x)\phi(y)\right]|0\rangle,\ \langle 0|T\left[\phi(x)\overline{\phi}(y)\right]|0\rangle.$$