Physics 722, Spring 2008
Problem Set 6: Renormalizability, Symmetries
Due Thursday, April 3.

1. *Renormalizability*

We argued that in four spacetime dimensions, a theory of spin-0 and spin-1/2 fields is generally not renormalizable if the Lagrangian contains operators of mass dimension > 4, and is renormalizable if it contains all operators of mass dimension ≤ 4 consistent with symmetries of the theory. What is the corresponding statement in d spacetime dimensions?

2. *Higgs Yukawa couplings*

Suppose $\Phi = (\phi_1^+, \phi_2^0)$ is a pair of complex scalar fields that transform as a doublet under an SU(2) symmetry. Suppose $\Psi_L = (\psi_{L1}^q, \psi_{L2}^{q-1})$ is a pair of fermions with left-handed chirality that transform as a doublet under the same SU(2) symmetry; and ψ_{R1}^q and ψ_{R2}^{q-1} are two right-handed SU(2) singlets. The superscripts stand for the electric charge.

a) Write down *all* renormalizable nonderivative interactions among these various fields that are invariant under the SU(2) symmetry and electromagnetism. Recall that if Φ transforms as an SU(2) doublet then so does $i\,\sigma^2\,\Phi^*$, but the electric charges change sign because of the complex conjugation. Remember that the interactions should be Hermitian and Lorentz invariant.

b) Replace Φ by $(0, v)$ in the cubic interactions you constructed in part (a), for some constant v. Expand those terms in terms of ψ_{Li} and ψ_{Ri}. You should find mass terms for all of the fermions.

c) Assume ψ_{R1} does not exist. Which fields are massless?

Comments:
The Higgs fields transform as a doublet under the $SU(2)_W$ gauge invariance of the Standard Model. It is assumed that the Higgs fields obtain a vacuum expectation value that can be chosen (by a gauge transformation) to take the form in part (b). As a result, the Yukawa couplings you constructed in part (a) give rise to Standard Model fermion masses.

If there is no right-handed neutrino, then the neutrino cannot get simple Yukawa couplings to the Higgs and the mass must come from elsewhere. Neutrinos can have Majorana masses without there being right-handed neutrinos, so despite the experimental evidence for neutrino masses it is not known whether or not right-handed neutrinos exist.