

## Physics 722, Spring 2007

Problem Set 3, due Thursday, February 21.

### 1. Nucleon Self Energy

Consider the theory of a “nucleon”  $\psi(x)$  with mass  $M$ , Yukawa coupled to a real scalar field  $\phi(x)$  with mass  $m$ :

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - M)\psi + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{m^2}{2}\phi^2 - g\bar{\psi}\psi\phi - \frac{\lambda}{4!}\phi^4 + \mathcal{L}_{\text{CT}}.$$

a) Calculate the one-loop renormalized nucleon self energy  $\widetilde{\Sigma}(\not{p})$ . The renormalized self energy should satisfy  $\widetilde{\Sigma}(M) = 0$  and  $d\widetilde{\Sigma}/d\not{p}|_{\not{p}=M} = 0$ . Use a hard momentum cutoff to regularize any divergent integrals appearing at intermediate stages of the calculation, and check that those divergences are cancelled in the renormalization procedure. Your result should be left in terms of integral(s) over a single Feynman parameter.

b) Repeat the calculation using regulator fields, and again using dimensional regularization, and show that divergences can be cancelled with counterterms of the same form as you used in part (a).

### 2. Gamma Matrices in Even Dimensions

In dimensional regularization we analytically continue in the number of spacetime dimensions. It is natural to consider what the gamma matrices look like in other than four (integer) dimensions.

In  $d$  spacetime dimensions, we want to find  $d$  matrices satisfying:

$$\begin{aligned}\gamma_0^\dagger &= \gamma_0, & \gamma_i^\dagger &= -\gamma_i, & i &= 1, \dots, d-1, \\ \{\gamma_\mu, \gamma_\nu\} &= 2g_{\mu\nu}, & \mu, \nu &= 0, \dots, d-1.\end{aligned}$$

It's easier to construct matrices satisfying,

$$\begin{aligned}\gamma_\mu^\dagger &= \gamma_\mu, & \mu &= 1, \dots, d, \\ \{\gamma_\mu, \gamma_\nu\} &= 2\delta_{\mu\nu}, & \mu, \nu &= 1, \dots, d.\end{aligned}$$

You can get the gamma matrices you want from these by letting  $\gamma_d \rightarrow \gamma_0$ ,  $\gamma_j \rightarrow i\gamma_j$ .

a) Assume  $d$  is even. Define,

$$a_1 = \frac{1}{2}(\gamma_1 + i\gamma_2),$$

$$\begin{aligned}
a_2 &= \frac{1}{2}(\gamma_3 + i\gamma_4), \\
&\vdots \\
a_{d/2} &= \frac{1}{2}(\gamma_{d-1} + i\gamma_d),
\end{aligned}$$

where the gamma matrices in these expressions satisfy the second set of conditions above.

Show that,

$$\begin{aligned}
\{a_i, a_j\} &= \{a_i^\dagger, a_j^\dagger\} = 0, \\
\{a_i, a_j^\dagger\} &= \delta_{ij} \quad i, j = 1, \dots, d/2.
\end{aligned}$$

This is the algebra of raising and lowering operators for  $d/2$  independent two-level systems.

- b) In two dimensions, construct a matrix representation for  $a$  and  $a^\dagger$ . What are  $\gamma_1$  and  $\gamma_2$  in that representation?
- c) In  $d$  even dimensions, what is the dimensionality of your representation of the gamma matrices? Evaluate  $\text{Tr } 1$  and  $\text{Tr } \not{p}\not{p}$  in that representation.
- d) Check that  $\prod_i \gamma_i$  anticommutes with all of the  $\gamma_i$ . This is the analog of  $\gamma_5$  in any even dimension.

**Comment:** In odd dimensions the first  $d - 1$  gamma matrices can be constructed as above, and  $\gamma_d = \pm\gamma_1\gamma_2\cdots\gamma_{d-1}$  completes the gamma matrix algebra. There are two independent representations of the gamma matrix algebra in odd dimensions, differing in the sign of  $\gamma_d$ . These representations are exchanged by parity, and both representations appear in a parity-conserving theory.