Physics 722, Spring 2008
Problem Set 1: Background Fields, Derivative Interactions
due Thursday, Jan 31.

1. Peskin & Schroeder, Problem 4.4

2. Consider a free real scalar field ϕ with mass m. The Lagrangian density is,

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)^2 - \frac{m_0^2}{2}\phi^2 .$$

If we define a rescaled field,

$$\tilde{\phi} = Z^{-1/2}\phi,$$

and let $m_0^2 = m^2 + \delta m^2$, then the Lagrangian density can be written as,

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \tilde{\phi}\right)^2 - \frac{m^2}{2}\tilde{\phi}^2 + \mathcal{L}_{int},$$

where

$$\mathcal{L}_{int} = (Z-1)\left[\frac{1}{2}\left(\partial_\mu \tilde{\phi}\right)^2 - \frac{m^2}{2}\tilde{\phi}^2\right] - Z\,\frac{\delta m^2}{2}\,\tilde{\phi}^2.$$

a) Using the naive Feynman rules for derivative interactions, and treating $\mathcal{L}_{int}$ as an interaction, what is the Feynman rule for the two-point vertex coming from $\mathcal{L}_{int}$?

b) Recall that the Feynman propagator for a real scalar field of mass m_0 is given by,

$$\int d^4x\, e^{ik\cdot x}\langle 0|T\left(\phi(x)\phi(0)\right)|0\rangle = \frac{i}{k^2 - m_0^2 + i\epsilon}.$$

The rescaled field $\tilde{\phi}$ should then satisfy,

$$\int d^4x\, e^{ik\cdot x}\langle 0|T\left(\tilde{\phi}(x)\tilde{\phi}(0)\right)|0\rangle = \frac{i\,Z^{-1}}{k^2 - m_0^2 + i\epsilon}.$$

Sum over all Feynman diagrams that contribute to $\langle 0|T\left(\tilde{\phi}(x)\tilde{\phi}(0)\right)|0\rangle$, and confirm the expected two-point function.