Physics 722, Spring 2007 Josh Erlich

Problem Set 2: Fermion Self Energy, due Tuesday, Feb 27.

1. *Nucleon Self Energy*

Consider the theory of a "nucleon" $\psi(x)$ with mass M, Yukawa coupled to a real pseudoscalar "pion" field $\phi(x)$ with mass m:

$$\mathcal{L} = \overline{\psi}(i\partial\!\!\!/ - M)\psi + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{m^2}{2}\phi^2 - ig\overline{\psi}\gamma^5\psi\phi - \frac{\lambda}{4!}\phi^4 + \mathcal{L}_{\rm CT}.$$

a) Calculate the one-loop renormalized nucleon self energy $\widetilde{\Sigma}(p\!\!\!/)$. The renormalized self energy should satisfy $\widetilde{\Sigma}(M) = 0$ and $d\widetilde{\Sigma}/dp\!\!\!/\,|_{p\!\!\!/=M} = 0$. Use a hard momentum cutoff to regularize any divergent integrals appearing at intermediate stages of the calculation, and check that those divergences are cancelled in the renormalization procedure. Your result should be left in terms of integral(s) over a single Feynman parameter.

b) Repeat the calculation using regulator fields, and again using dimensional regularization, and show that divergences can be cancelled with counterterms of the same form as you used in part (a). When using dimensional regularization, you can assume $\{\gamma^5, \gamma^\mu\} = 0$, just as in four dimensions.

c) The nucleon self energy has a branch cut beginning at some value of p^2. Identify that value of p^2 and give its physical interpretation.