Physics 722, Spring 2007 Josh Erlich

Problem Set 2: Scalar Self Energy, due Tuesday, Feb 20.

1. *Derivative Interactions*

In class we pointed out the subtleties in dealing with derivative interactions. Here you will study a simple example which demonstrates that the naive handling of derivatives gives the correct answer.

Consider a free real scalar field with Lagrangian,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2.$$

We can equivalently write this as a Lagrangian in terms of the rescaled field $\tilde{\phi} \equiv Z^{-1/2}\phi$ as,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \tilde{\phi})^2 - \frac{m^2}{2}\tilde{\phi}^2 + (Z-1)\left[\frac{1}{2}(\partial_\mu \tilde{\phi})^2 - \frac{m^2}{2}\tilde{\phi}^2\right].$$

Since we know the relation between ϕ and $\tilde{\phi}$, we also know the relation between two-point functions:

$$\langle 0|T\left(\tilde{\phi}(x)\tilde{\phi}(0)\right)|0\rangle = Z^{-1}\langle 0|T\left(\phi(x)\phi(0)\right)|0\rangle.$$

In terms of its Fourier transform, we then have.

$$\int \frac{d^4k}{(2\pi)^4}\, e^{ik\cdot x}\langle 0|T\left(\tilde{\phi}(x)\tilde{\phi}(0)\right)|0\rangle = \frac{i\,Z^{-1}}{k^2 - m^2 + i\epsilon}$$

You are to check this result by evluating $\langle 0|T\left(\tilde{\phi}(x)\tilde{\phi}(0)\right)|0\rangle$ in perturbation theory, where you are to think of the the terms in the Lagrangian proportional to $(Z-1)$ as being interaction terms.

a) What are the Feynman rules in this theory, treating the derivatives in the interaction naively?

b) By summing over *all* diagrams that contribute to the Fourier transformed two-point function, $\int \frac{d^4k}{(2\pi)^4}\, e^{ik\cdot x}\langle 0|T\left(\tilde{\phi}(x)\tilde{\phi}(0)\right)|0\rangle$, show that you recover the expected result.

2. *Scalar Self Energy*

Consider the theory of a single real scalar field,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 - \frac{g_3}{3!}\,\phi^3 - \frac{g_4}{4!}\,\phi^4.$$

Calculate the one-loop renormalized self energy $\widetilde{\Pi}(k^2)$ for the scalar field ϕ. $\widetilde{\Pi}(k^2)$ should satisfy the renormalization conditions $\widetilde{\Pi}(m^2) = 0$ and $d\widetilde{\Pi}/dk^2|_{k^2=m^2} = 0$. Your result should be left in terms of integral(s) over a single Feynman parameter.