Problem Set 8: Functional integral and gauge invariance
Due Tuesday, April 25.

1. *Coulomb gauge*

Coulomb gauge is defined by the condition $\nabla \cdot \mathbf{A} = 0$. The Fadeev-Popov determinant is a constant in this case, as in the covariant gauges discussed in class. The determinant can then be absorbed in the normalization of the functional integral.

a) Show that the spatial components of the photon propagator are,

$$\frac{-i}{k^2 + i\epsilon}\left(g_{ij} + \frac{k_i k_j}{\mathbf{k}^2}\right).$$

b) Show that the 0-0 component of the photon propagator is,

$$\frac{i}{\mathbf{k}^2}.$$

c) What are the time-space components of the photon propagator?

2. *Gauge invariance of Feynman amplitudes*

Calculate the $\mathcal{O}(e^2)$ Feynman amplitude for electron-electron scattering in Landau gauge and in Feynman gauge. Show that the result is the same. Do you need to use the fact that the electrons are on-shell?