Problem Set 7: Renormalization Group

Due Tuesday, April 18.

1. *Long Range Interactions*

Long range interactions between spins can be described by a term in the Landau-Ginzburg Hamiltonian of the form,

$$\beta H \supset \int d^d x \int d^d y \; J(|\mathbf{x}-\mathbf{y}|)\, m(\mathbf{x}) \cdot m(\mathbf{y}).$$

For concreteness, consider a long ranged interaction governed by a term of the above form with $J(r) \propto 1/r^{d+\sigma}$.

Fourier transforming this and adding to the rest of the Fourier transformed Hamiltonian, we get an expansion of the form,

$$\beta H = \int \frac{d^d q}{(2\pi)^d} \frac{t + K_2\, q^2 + K_\sigma\, q^\sigma + \cdots}{2} m(\mathbf{q}) \cdot m(-\mathbf{q})$$

$$+u \int \frac{d^d q_1\, d^d q_2\, d^d q_3}{(2\pi)^{3d}}\, m(\mathbf{q}_1) \cdot m(\mathbf{q_2})\, m(\mathbf{q}_3) \cdot m(-\mathbf{q}_1 - \mathbf{q}_2 - \mathbf{q}_3).$$

(Can you show this?)

a) For $u = 0$, construct the recursion relation for the evolution of the parameters (t, K_2, K_σ) under the renormalization group transformation.

b) Fix the contrast factor which rescales the magnetization $m(\mathbf{q})$ so that K_2 is unchanged under the RG.

c) Show that K_σ is irrelevant if $u = 0$ and $\sigma > 2$.

d) What is the fixed point Hamiltonian in this case?

e) Plot the RG flows in the (t, K_σ) plane for $\sigma > 2$ and for $\sigma < 2$.