Problem Set 4: Electron Vertex Function, Mean Field Approximation

Due Tuesday, March 28.

1. *Rosenbluth Formula*

This is problem 6.1 in Peskin and Schroeder.

The vertex function for some Dirac spinor field is of the form,

$$\tilde{\Gamma}(p', p) = \gamma^\mu F_1(q^2) + \frac{i\,\sigma^{\mu\nu}\,q_\nu}{2m} F_2(q^2),$$

where p, p' are the ingoing, outgoing fermion momenta, and $q = p' - p$ is the ingoing photon momentum, and $\sigma^{\mu\nu} = i/2\,[\gamma^\mu, \gamma^\nu]$. (A factor of electric charge e is factored out of $\tilde{\Gamma}$ as for the electron.)

The form factors for strongly interacting particles like the proton are generally difficult to calculate, but they can be determined experimentally. Consider scattering of an energetic electron with energy $E \gg m_e$ from a proton initially at rest. To leading order in e the electron vertex function can be approximated by the tree level interaction vertex, and the proton electromagnetic form factors $F_1(q^2)$ and $F_2(q^2)$ contain information about the strong interactions.

At leading order in $\alpha = e^2/4\pi$, show that the elastic scattering cross section takes the Rosenbluth form,

$$\frac{d\sigma}{d\,\cos\theta} = \frac{\pi\alpha^2\left[\left(F_1^2 - \frac{q^2}{4m^2}F_2^2\right)\cos^2\frac{\theta}{2} - \frac{q^2}{2m^2}\left(F_1 + F_2\right)^2\sin^2\frac{\theta}{2}\right]}{2E^2\left[1 + \frac{2E}{m}\sin^2\frac{\theta}{2}\right]\sin^4\frac{\theta}{2}},$$

where θ is the lab frame scattering angle and F_1 and F_2 are evaluated at the q^2 associated with elastic scattering at this angle.

2. *Tricritical Point*

Consider the Landau-Ginzburg Hamiltonian,

$$\beta H = \int d^d x \left[\frac{K}{2}(\nabla m)^2 + \frac{t}{2}m^2 + u\,m^4 + v\,m^6 - h\,m\right].$$

If $u < 0$ then the m^6 term is important for stability.

that in the saddle point approximation to the functional integral describing the partition function, there is a first order phase transition for which the magnetization m jumps discontinuously.

b) Calculate $\bar{t}$ and $\bar{m}$ at this transition.

c) For $h = 0$ and $v > 0$ plot the phase boundary in the (u, t) plane. Identify the phases and the order of the phase transitions.

d) The point $u = t = 0$ separates first and second order phase boundaries. It is called a *tricritical point.* For $u = 0$ calculate the mean field critical exponents α, β, γ and δ, where $C \sim |t|^{-\alpha}$, $\bar{m}(h = 0) \sim |t|^{\beta}$, $\chi \sim |t|^{-\gamma}$, and $\bar{m}(t = 0) \sim h^{1/\delta}$.