Problem Set 4: Electron Vertex Function
Due Thursday, March 16.

In this problem set you will work through most of the calculation of the renormalized one-loop electron vertex function in QED. *Warning:* It is a bit of work, but you have two weeks to do it. This calculation is done in every field theory textbook, but try to do it on your own. If you get stuck, use the books or ask me.

You should assume that the electrons are on-shell, but not the photon. You can use relations that are valid when the vertex is sandwiched between $\overline{u}(p')$ on the left and $u(p)$ on the right. For example, $\not{p}$ on the right can be replaced by m, as can $\not{p}'$ on the left.

1. Using the Feynman rules for QED write out the one-loop contribution to the renormalized electron vertex function as an integral over the loop momentum.

2. Combine denominators using Feynman's trick and express the one-loop vertex function as an integral over the loop momentum and Feynman parameters. Indicate the integration region for the Feynman parameters.

3. Complete the square to make the integrand invariant under Lorentz transformations of the shifted loop momentum.

4. Due to the Lorentz symmetry, the following identities hold:

$$\int \frac{d^4k}{(2\pi)^4}\, k^\mu\, f(k^2) = 0,$$

$$\int \frac{d^4k}{(2\pi)^4}\, k^\mu k^\nu\, f(k^2) = \int \frac{d^4k}{(2\pi)^4}\, g^{\mu\nu}\, k^2\, f(k^2)/4.$$

Use these relations to simplify your expression for the one-loop vertex function.

5. Use either Pauli-Villars (and our integral table) or dimensional regularization (and our other integral table) to regulate the integral over the shifted loop momentum. *Hint*: If you use Pauli-Villars regularization a single photon regulator field will do the job here. For these purposes, the

$$\frac{+i\, g^{\mu\nu}}{k^2 - M^2 + i\epsilon},$$

where M is the regulator field mass.

6. Your result is probably not in the desired form,

$$\tilde{\Gamma}^\mu(p', p) = e\gamma^\mu\, F_1(q^2) + \frac{ie\, \sigma^{\mu\nu} q_\nu}{2m}\, F_2(q^2).$$

Become friends with the gamma matrices if you are not already, and manipulate the numerator in the integral to put it in the desired form. Identify $F_1(q^2)$ and $F_2(q^2)$. This will probably be the bulk of your work in this problem set.

7. Is the one-loop contribution to $F_1(q^2)$ divergent? How about $F_2(q^2)$? Explain why the nondivergent part had to be that way, arguing based on renormalizability of QED.