Problem Set 2: Self-Energy and Dimensional Regularization
Due Thursday, February 23.

1. *Meson Self Energy*

Consider the pseudoscalar meson-nucleon theory, with interaction $\mathcal{L}_I = g\overline{N}i\gamma^5 N\phi$. In the previous problem set you calculated the renormalized one-loop meson self energy,

$$\widetilde{\Pi}(k^2) = \Pi(k^2) - \mathrm{Re}\left[\Pi(\mu^2)\right] - \mathrm{Re}\left[\Pi'(\mu^2)\right]\ (k^2 - \mu^2).$$

When $k^2 > 4m^2$ the self-energy is imaginary. What is the sign of the imaginary part? Explain why the sign is consistent with the general analytic properties of the renormalized propagator.

2. *Renormalization and Dimensional Regularization*

Consider a real scalar field ϕ with Lagrangian density,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{1}{6}g\phi^3.$$

Ignore the problem that this theory is unstable because the potential is unbounded below.

What is the maximum number of dimensions for which this theory is renormalizable?

In 6 dimensions, calculate the 1-loop diagram contributing to the self-energy of the scalar field. Use dimensional regularization to identify the divergences as $d \to 6$.

Still in six dimensions, calculate the renormalized self energy satisfying $\widetilde{\Pi}(\mu^2) = 0$ and $\widetilde{\Pi}'(\mu^2) = 0$.