1. **Nucleon Self Energy**

Consider the pseudoscalar meson-nucleon theory, with interaction $\mathcal{L}_I = g N^i \gamma^5 N \phi$.

Calculate the renormalized one-loop nucleon self energy $\widetilde{\Sigma}(p)$ in this theory if the physical nucleon mass is $m$ and the physical meson mass is $\mu$. Express your answer in the form of an integral over a single Feynman parameter.

The renormalized propagator has a branch cut. Identify the value of $p^2$ at the beginning of the branch cut.

What is the meaning of the position of the branch point? *(Hint: Recall that we argued that the spectral function has a branch cut due to the continuum of intermediate multiparticle states.)*

2. **Meson Self Energy**

In the same theory, calculate the renormalized one-loop meson self energy $\widetilde{\Pi}(k^2)$ satisfying the renormalization conditions $\widetilde{\Pi}(\mu^2) = 0$ and $\widetilde{\Pi}'(\mu^2) = 0$.

You can use the counterterms $\mathcal{L}_{ct} = a(\partial_\mu \phi)^2 - b \phi^2$ to satisfy the renormalization conditions by choosing:

$$\widetilde{\Pi}(k^2) = \Pi(k^2) - \Pi(\mu^2) - \Pi'(\mu^2)(k^2 - \mu^2).$$

Identify the beginning of the branch cut as a function of $k^2$. What is the interpretation of the location of the branch point?

3. **Renormalizability**

We argued that in four spacetime dimensions, a theory of spin-0 and spin-1/2 fields is generally not renormalizable if the Lagrangian contains operators of mass dimension $> 4$, and is renormalizable if it contains all operators of mass dimension $\leq 4$. What is the corresponding statement in $d$ spacetime dimensions?