Problem Set 1: Self Energies
Due Thursday, February 16.

1. *Nucleon Self Energy*

Consider the pseudoscalar meson-nucleon theory, with interaction $\mathcal{L}_I = g\overline{N}i\gamma^5 N\phi$.

Calculate the renormalized one-loop nucleon self energy $\widetilde{\Sigma}(\not{p})$ in this theory if the physical nucleon mass is m and the physical meson mass is μ. Express your answer in the form of an integral over a single Feynman parameter.

The renormalized propagator has a branch cut. Identify the value of p^2 at the beginning of the branch cut.

What is the meaning of the position of the branch point? (*Hint:* Recall that we argued that the spectral function has a branch cut due to the continuum of intermediate multiparticle states.)

2. *Meson Self Energy*

In the same theory, calculate the renormalized one-loop meson self energy $\widetilde{\Pi}(k^2)$ satisfying the renormalization conditions $\widetilde{\Pi}(\mu^2) = 0$ and $\widetilde{\Pi}'(\mu^2) = 0$.

You can use the counterterms $\mathcal{L}_{ct} = a(\partial_\mu\phi)^2 - b\phi^2$ to satisfy the renormalization conditions by choosing:

$$\widetilde{\Pi}(k^2) = \Pi(k^2) - \Pi(\mu^2) - \Pi'(\mu^2)\,(k^2 - \mu^2).$$

Identify the beginning of the branch cut as a function of k^2. What is the interpretation of the location of the branch point?

3. *Renormalizability*

We argued that in four spacetime dimensions, a theory of spin-0 and spin-1/2 fields is generally not renormalizable if the Lagrangian contains operators of mass dimension > 4, and is renormalizable if it contains all operators of mass dimension ≤ 4. What is the corresponding statement in d spacetime dimensions?