Physics 721, Fall 2023
Problem Set 8
Due Monday, November 6

1. Prove Wick's theorem for fermions: For any free fermionic fields $A_i(x_i)$,

$$T[A_1(x_1)\,A_2(x_2)\cdots A_n(x_n)] \;=\; :A_1(x_1)\cdots A_n(x_n): \\ \qquad\qquad + \text{ all normal-ordered contractions.}$$

The contractions are defined as in the bosonic case:

$$\overbracket{A(x)B}(y) = T[A(x)B(y)] - :A(x)B(y):$$

The sign rules for exchange of fermionic fields are as follows:

$T[A(x)B(y)C(z)] = -T[A(x)C(z)B(y)]$, *i.e.* a minus sign for each exchange of neighboring fermion fields under the time-ordering symbol.

$:A(x)B(y)C(z): = -:A(x)C(z)B(y):$, *i.e.* a minus sign for each exchange of neighboring fermion fields under the normal-ordering symbol.

$:\overbracket{A(x)B(y)C}(z)D(w): = -\overbracket{A(x)C}(z)\;:B(y)D(w):$, *i.e.* a minus sign for each exchange of neighboring fermion fields required to make contracted fields neighbors.

2. a) Using Wick's theorem evaluate the following correlation functions of free real scalar field $\phi(x)$, in terms of Feynman propagators $\Delta_F(x-y) = \langle 0|T\left[\phi(x)\phi(y)\right]|0\rangle = \overbracket{\phi(x)\phi}(y)$.

$$\langle 0|T\left[\phi(x_1)\phi(x_2)\phi(x_3)\right]|0\rangle$$
$$\langle 0|T\left[\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\right]|0\rangle$$

b) Using Wick's theorem evaluate the following correlation functions of free complex scalar field $\phi(x)$, in terms of Feynman propagators $\Delta_F(x-y)$:

$$\langle 0|T\left[\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\right]|0\rangle$$
$$\langle 0|T\left[\phi(x_1)\phi(x_2)\phi(x_3)^\dagger\phi(x_4)^\dagger\right]|0\rangle$$