**Physics 721, Fall 2023**
**Problem Set 5**
Due Monday, October, 16.

1. *Chirality*

Any Dirac spinor can be decomposed into a left-handed and a right-handed
part by using the **chirality projection operators**,

$$P_L = \frac{1 - \gamma^5}{2}, \quad P_R = \frac{1 + \gamma^5}{2}.$$

Using the properties of $\gamma^5$ show that:

a) $P_L^2 = P_L, \quad P_R^2 = P_R, \quad P_L P_R = P_R P_L = 0.$

b) Given a Dirac spinor $\psi$ define its left-handed part as $\psi_L \equiv P_L \psi$ and
its right-handed part as $\psi_R \equiv P_R \psi$. Show that under a Lorentz transfor-
mation of $\psi$, the left and right-handed components of $\psi$ transform inde-
pendently. This implies that the Dirac spinor forms a **reducible repre-
sentation** of the Lorentz group.

c) By acting on the Dirac equation $(i\partial\!\!\!/ - m)\,\psi = 0$ with $P_L$ and with $P_R$,
rewrite the Dirac equation in terms of a coupled set of equations for $\psi_L$
and $\psi_R$.

d) Show that the equations for $\psi_L$ and $\psi_R$ decouple when $m \to 0$.


2. *Majorana spinor fields*

Peskin & Schroeder, Problem 3.4a,b.