Physics 721, Fall 2023
Problem Set 10 Due Monday, December 4.

1. *If the Photon Were a Scalar*

Imagine that electrons ψ_e and muons ψ_μ coupled to a massless scalar field ϕ instead of a massless photon, via an interaction term in the Lagrangian,

$$\mathcal{L} \supset -e\,\overline{\psi}_e \psi_e \phi - e\,\overline{\psi}_\mu \psi_\mu \phi.$$

a) What are the Feynman rules for this theory, *i.e.* the rules for vertices, propagators, and external lines? Keep in mind that $\psi_e(x)$ and $\psi_\mu(x)$ anti-commute with one another, so that contractions between different fermion types vanish. Furthermore, there are two types of vertices in this theory: one involving the electron field and one involving the muon field.

b) Calculate the differential cross section $d\sigma/d\Omega$ for unpolarized $e^+ + e^- \to \mu^+ + \mu^-$ scattering to lowest nontrivial order in the coupling e, summed over final-state muon and antimuon spins and averaged over initial-state electron and positron spins. Since the muon is around 200 times heavier than the electron, you may simplify your calculation by approximating the electron as being massless.

You should express your result in terms of the center-of-mass electron energy E, the muon mass m_μ, and the electromagnetic coupling $\alpha = e^2/(4\pi)$.

You may find it useful to follow the calculation of the cross section for the same reaction in QED, as in Peskin & Schroeder, section 5.1 and as in the course notes. In particular, note that only one Feynman diagram contributes at this order, not two as in the case of $e^+e^- \to e^+e^-$ scattering.

c) By integrating over solid angle Ω, calculate the total cross section σ_{Tot}.

d) By measuring the differential cross section, could you distinguish this theory from Quantum Electrodynamics?

2. *Decay of a scalar particle*
(Optional - won't be graded, but you should understand how to solve this problem.)

Consider a theory of three real scalar fields $A(x),\ B(x),\ C(x)$, with Lagrangian,

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\left[(\partial_\mu A)^2 - M^2\,A^2 + (\partial_\mu B)^2 - m^2\,B^2 + (\partial_\mu C)^2 - m^2\,C^2\right] \\ &\quad - \left(g_1\,A^3 + g_2\,AB^2 + g_3\,ABC\right),\end{aligned}$$

where $M > 2m$, and the couplings g_1, g_2, and g_3 are real.

a) At $\mathcal{O}(g_i^2)$ in the decay rates, what sets of particles can A decay into?

b) At the same order in g_i, what is the ratio,

$$\frac{\Gamma(A \to BC)}{\Gamma(A \to BB)}\ ?$$

Be careful to sum over all contributions to the invariant scattering amplitude at lowest order in g_i. It may be useful to begin with the relevant terms in the expansion of the time-ordered exponential.

c) What is the total decay rate $\Gamma(A \to \text{anything})$ in terms of M, m and g_i at $\mathcal{O}(g_i^2)$?

d) What is the total decay rate for the B particle, $\Gamma(B \to \text{anything})$?