The positive frequency solutions to the Dirac equation can be written in a form valid for arbitrary 3-momentum:

$\Psi(x) = u(p) e^{-ip\cdot x}$, $p^\mu p_\mu = m^2$, $p^0 > 0$, with two linearly independent solutions for $u(p)$:

$$\boxed{u^s(p) = \begin{pmatrix} \sqrt{p^\mu \sigma_\mu}\, \xi^s \\ \sqrt{p^\mu \bar{\sigma}_\mu}\, \xi^s \end{pmatrix}} \quad , s = 1, 2$$

$$\sigma^\mu = (1, \vec{\sigma})^\mu, \quad \bar{\sigma}^\mu = (1, -\vec{\sigma})^\mu,$$

$$\xi^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \xi^2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Normalization: $u^{r\dagger}(p) u^s(p) = \xi^{r\dagger} p^\mu \sigma_\mu \xi^s + \xi^{r\dagger} p^\mu \bar{\sigma}_\mu \xi^s$
$= 2p^0 \delta^{rs} \equiv 2\omega_{\vec{p}} \delta^{rs}$

where $\omega_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$

Equivalently, $\bar{u}^r(p) u^s(p) = \xi^{r\dagger} (p^\mu \sigma_\mu)^{1/2} (p^\nu \bar{\sigma}_\nu)^{1/2} \xi^s$
$+ \xi^{r\dagger} (p^\mu \bar{\sigma}_\mu)^{1/2} (p^\nu \sigma_\nu)^{1/2} \xi^s$
$= 2\, \xi^{r\dagger} \left((p^0)^2 - \sum_{ij} p^i p^j \sigma^i \sigma^j \right)^{1/2} \xi^s$
$= 2\, \xi^{r\dagger} \left((p^0)^2 - \vec{p}^2 \right)^{1/2} \xi^s$
$= 2m\, \delta^{rs}$

$$\rightarrow \boxed{\bar{u}^r(p) u^s(p) = 2m\, \delta^{rs}}$$

Negative-Frequency Solutions

$\psi(x) = v(p)e^{ip\cdot x}$, $p^\mu p_\mu = m^2$, $p^0 > 0$ (but note positive sign in exponent)

Rest frame: $p^0 = m$, $\vec{p} = 0$.
Dirac equation: $-(\gamma^\mu p_\mu + m)v(p) = 0$

$$-m\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} v(p) = 0$$

Solutions: $v(p) = \sqrt{m}\begin{pmatrix} \eta \\ -\eta \end{pmatrix}$, η = constant 2-component spinor

In the Weyl basis the Lorentz transformations act independently on the top two and bottom two components, so we can read off the boosted solutions from the positive-frequency solutions.

$$\rightarrow \boxed{v^s(p) = \begin{pmatrix} \sqrt{p^\mu \sigma_\mu}\ \eta^s \\ -\sqrt{p^\mu \bar{\sigma}_\mu}\ \eta^s \end{pmatrix}}, \quad s = 1,2, \quad \eta^s = \text{basis of 2-component spinors.}$$

Normalization: $v^{r\dagger}(p)v^s(p) = 2\omega_{\vec{p}}\,\delta^{rs}$
Equivalently $\boxed{\bar{v}^r(p)\,v^s(p) = -2m\,\delta^{rs}}$

Furthermore, the positive and negative-frequency solutions are orthogonal in the sense that

$$\boxed{\bar{u}^r(p)v^s(p) = \bar{v}^r(p)u^s(p) = 0}$$