Physics 721, Fall 2006 Josh Erlich

Problem Set 7: Decays, Scattering and Identical Particles

Due Thursday, December 7.

1. *Scalar Particle Decays*

Consider a theory of three real scalar fields $A,\ B,\ C$, with Lagrangian,

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\left[(\partial_\mu A)^2 - M^2\,A^2 + (\partial_\mu B)^2 - m^2\,B^2 + (\partial_\mu C)^2 - m^2\,C^2\right] \\ &\quad -g\left(A^3 + AB^2 + ABC\right),\end{aligned}$$

where $M > 2m$.

a) At $\mathcal{O}(g^2)$ in the decay rates, what sets of particles can A decay into?

b) At the same order in g, what is the ratio,

$$\frac{\Gamma(A \to BC)}{\Gamma(A \to BB)}\ ?$$

Be careful to sum over all contributions to the invariant scattering amplitude at lowest order in g. It may be useful to begin with the relevant terms in the expansion of the time-ordered exponential.

c) What is the total decay rate $\Gamma(A \to \text{anything})$ in terms of M, m and g at $\mathcal{O}(g^2)$?

d) What is the total decay rate for the B particle, $\Gamma(B \to \text{anything})$?

2. *Pauli Exclusion Principle in Scattering Amplitudes*

Consider elastic electron scattering $e^- + e^- \to e^- + e^-$ in QED. Show that the scattering amplitude to $\mathcal{O}(e^2)$ vanishes if the final state electrons have the same quantum numbers. This is a demonstration that our formalism has the Pauli exclusion principle for fermions built in: you will never create a state in which identical fermions have the same quantum numbers.