Physics 721, Fall 2006 Josh Erlich

Problem Set 4: The Feynman Propagator, Dirac Fields

Due Tuesday, October 23.

1. *Propagators for the Complex Scalar Field*

Consider a free complex scalar field $\phi(x)$ with Lagrangian,

$$\mathcal{L} = |\partial_\mu \phi|^2 - m^2 |\phi|^2.$$

a) Using only the equations of motion and the equal time commutation relations, show that for a complex scalar with mass m,

$$\left(\partial_\mu \partial^\mu + m^2\right) \langle 0| T\left[\phi(x)\phi^\dagger(y)\right]|0\rangle = -i\,\delta^4(x-y),$$

where the derivatives are with respect to the coordinates x, and T is the time ordering symbol.

Hence, $i\langle 0|T\left[\phi(x)\phi^\dagger(y)\right]|0\rangle$ is a Green's function for the Klein-Gordon equation.

b) Decompose $\phi(x)$ as,

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3\sqrt{2\omega_{\mathbf{k}}}} \left(a_{\mathbf{k}}\, e^{-ik\cdot x} + b^\dagger_{\mathbf{k}}\, e^{ik\cdot x}\right),$$

where $k^0 = \omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + m^2}$. Using the harmonic oscillator commutation relations, calculate:

$\langle 0|T\left[\phi(x)\phi(y)\right]|0\rangle$, $\langle 0|T\left[\phi^\dagger(x)\phi^\dagger(y)\right]|0\rangle$, and $\langle 0|T\left[\phi(x)\phi^\dagger(y)\right]|0\rangle$.

2. *Spatial Momentum in the Dirac Field*

Consider a free Dirac spinor field with mass m.

a) What is the conserved energy-momentum tensor $T^{\mu\nu}$ in terms of $\psi(x)$?

b) Using the plane wave decomposition of $\psi(x)$ and the harmonic oscillator anticommutation relations, calculate the normal-ordered spatial momentum in the Dirac field in terms of a single d^3k integral involving the creation and annihilation operators.

c) Explain why $a^{r\,\dagger}_{\mathbf{k}}$ and $b^{r\,\dagger}_{\mathbf{k}}$ create particles with momentum $\mathbf{k}$.